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The transition to equilibrium in a system with gravitationally interacting particles. II. Gravitational instability

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Abstract The distribution function of systems in equilibrium must have the canonical form of the Gibbs distribution. Attempts have been made for more than 100 years to substantiate this behaviour of systems to involve their mechanical behaviour. In other words, it seems that a huge number of particles of the medium, resulting from interaction with each other according to dynamic laws, can explain the statistical behaviour of systems during their transition to equilibrium. Modelling of gravitationally interacting particles is carried out, and it is shown that, in this case, the distribution function does not evolve to the canonical form. Earlier, the same results were obtained for classical Coulomb plasma. On the other hand, such a statistical effect as relaxation is well described by the system's dynamic behaviour, and the simulation data agree with the known theoretical results obtained in various statistical approaches. This article demonstrates that the well-known phenomenon of gravitational instability is also reproduced in the numerical simulation of a system with gravitationally interacting particles.

Keywords: *dynamical behaviour in classical mechanics, substantiation of statistical mechanics, entropy, equilibrium, gravitationally interacting particles.*

1 Introduction

The formulation of the dynamical laws of macroscopic bodies, given by I. Newton, led to successes in quantitatively describing their behaviour. The discovery of the planets of the solar system Neptune (according to the calculations of W.J. Le Verrier and D.C. Adams) and Pluto (according to the calculations of P. Lovell and W.G. Pickering) clearly confirm this. We can predict the positions of the planets for centuries. Methods for describing the mechanical behaviour of systems are constantly being improved [1-12]. However, with increased bodies in the system, predicting their behaviour using Newton's laws becomes much more difficult. Such a prediction becomes impossible for systems with many particles, such as gases, liquids, and solids. In this case, statistical methods of description are used. But we, on the one hand, understand that this huge number of particles must still be described by the laws of dynamics. On the other hand, when trying to do this, we encounter problems that, at first glance, should not arise. The issues of substantiating the statistical description and their connection with the dynamic description have become essential and come to the fore in science since the beginning of the 20th century [13].

If a closed system at a certain moment of time is in a non-equilibrium macroscopic state, then the most probable consequence at subsequent moments of time will be a monotonic increase in the system's entropy. This is the so-called law of increasing entropy or the second law of thermodynamics. It was discovered by R. Clausius, and its statistical justification was given by L. Boltzmann [14-16].

Systems in equilibrium. Conclusions about the increase in entropy in a closed system and the form of the distribution function of systems in equilibrium can be traced in many available monographs. The conclusion that entropy increases (or at least does not change) during an irreversible transition from one equilibrium state to another is proved, for example, in [14] using the postulate that the second kind of perpetuum mobile is impossible. The derivation of the distribution function for closed systems in equilibrium, based on the microcanonical distribution, is contained, for example, in [15]. The distribution function $w(E)$ of systems with energy E in equilibrium depends exponentially on the entropy of the system S,

$$w(E) \propto \exp\,(S(E)),$$

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which is why it is stated that the entropy of a closed system in a state of complete statistical equilibrium has the largest possible value (for a given energy of the system). The form of the distribution function of a system as a function of its entropy is rarely used. Most often, depending on the consideration being carried out, its equivalent representations are used either in the form of a Gibbs distribution (canonical distribution)

$$w(E) \propto \exp\left(-E/T\right),$$

where T is the system's temperature or in the form of a Boltzmann distribution. It is generally assumed that when an equilibrium is established in the energy ranges, where the interaction of particles with other particles can be neglected or the interaction of a particle with only the nearest particle can be taken into account, the Gibbs distribution function transforms into the Boltzmann distribution function of particles.

The use of the above-mentioned equilibrium distribution functions has been carried out in consideration of a huge variety of practical problems, so there is no doubt that they are confirmed by all our daily observations. But not everything is as simple as it seems at first glance. The fact is that when considering the world around us as a whole, it is impossible not to notice that with the destruction of some systems, which just corresponds to the growth of the entropy of such systems, the organization or ordering of other systems takes place. The emergence of man is a vivid confirmation of this. However, the processes of structuring and complication of systems lead not to an increase but to a decrease in entropy. To a certain extent, these emerging difficulties have not yet been overcome.

The distribution function of particles is formed due to their interaction during the transition to an equilibrium state. That is why it is natural to assume that the statistical behaviour of the particles must be ultimately described by their dynamic behaviour. It is precisely under the excitation, or when external conditions are changed due to the laws of dynamics, that the interacting particles should line up and redistribute in such a way as to reproduce their distribution function. Everything seems to be logical. However, more than a century of attempts to substantiate this statement did not lead to success. A detailed description of these attempts is given in our previous work [17]. It describes the main milestones on the way to substantiate this position, starting with the Boltzmann hypothesis of a giant fluctuation, involving the explanation of open systems, Gibbs's steps to introduce coarsening of the phase liquid, his introduction of the concept of mixing and further development of this concept in the works of Krylov, Sinai, Kolmogorov and his school, applying Poincare and his followers to questions of regularity and stochasticity of non-integrable systems, attempts to justify the ergodicity of statistical systems, including consideration of ergodicity and quasi-ergodicity in the context of metric transitivity and mixing, consideration of questions about the instability of phase trajectories with the involvement of Lyapunov exponents and ending with the contribution of synergetics to the possible resolution of this issue.

Because the question of substantiating the statistical behaviour of the system by its dynamic behaviour could not be solved, various questions of physics needed to proceed to the justification of various scales (or arrows) of time, among which the main ones were considered, i.e. the thermodynamic scale, cosmological, causal, psychological, and quantum. If we abstract from some details, all these time scales are determined by the thermodynamic scale [17], returning us to the starting point – the need to explain the irreversible growth of entropy in closed mechanical systems.

Why, then, has the justification mentioned above defied the explanation for more than 100 years (it is believed that serious research in this area began after the publication of the review [13])? Aside from considering particle interaction dynamics in a closed system, there must be something else that is not apparent on the surface. It seems that the entropy during the transition to equilibrium should increase. However, as noted above, it can also decrease, which we observe in the example of highly organized systems. However, generally speaking, it should not change. There are rigorously proven theorems on the conservation of entropy in closed mechanical systems, both classical and quantum [18-20]. In addition, the particle motion equations are reversible in time, which should not help explain the irreversible processes of entropy growth.

To clarify this issue, attempts were made to numerically simulate the interaction of particles based on their dynamic behaviour by Newton's laws. This approach has been developed for a long time with various physical issues, i.e. the so-called particle-in-cell simulations [21]. A classical Coulomb plasma was modelled in works [22-27]. Initially, the simulation aimed to refine the rate constant of triple recombination of ions with electrons by ab initio methods. The rate constant was necessary for modelling the kinetics of plasma processes, particularly in questions concerning the theory of plasma lasers and their achievable characteristics [28-36]. However, research very quickly reached the level of fundamental questions of plasma physics and statistical physics. It turned out that the modelled plasma "did not want" to recombine under the known concepts of the recombination process. More precisely, it did not recombine at all. Further, with a more detailed study of this issue, it turned out that the form of the distribution function of plasma particles after establishing equilibrium in numerical simulations is neither Boltzmann nor microcanonical. The recombination of the simulated plasma began only when the dynamic memory of the system disappeared in one way or another. This was achieved in modelling by introducing various kinds of stochastics; for example, the particle velocities were forcibly rearranged arbitrarily while maintaining their total energy, etc. (for more details, see [17, 22-27]).

In our previous work [17], the simulation was carried out in a system of gravitationally interacting particles. Although the system of gravitationally interacting particles and a system of interacting charged particles seem similar due to the identical behaviour of the interaction force on distance, they have significant distinctions (for more details, see [17]). It turned out that when equilibrium is established, the distribution function of particles, as well as in plasma modelling, is neither Boltzmann nor microcanonical.

So, according to well-known concepts, the form of the distribution function particles that are in equilibrium in closed systems must have the canonical Gibbs form. On the other hand, according to known theorems of mechanics, it cannot have this form since the entropy of a system cannot change and reach its maximum value in equilibrium. The numerical simulations carried out in the above-mentioned works show that the latter case is actually implemented if nothing but the numerical solution of the equations of particle motion according to Newton's laws is used.

In the work [17], numerical methods were used to investigate the distribution function of gravitationally interacting particles and their relaxation time. Even though during the transition to equilibrium, the particle distribution function for the total energy did not take the canonical Gibbs form, the relaxation time, nevertheless, turned out to be in full agreement with the known values obtained by various methods, ranging from simple estimates to its consideration it in the framework of the Fokker-Planck approximation and kinetic description. This paper aims to study gravitational instability within the framework of the same numerical simulation of the behaviour of particles interacting with each other through the law of gravity. This method is described in more detail in our previous work [17].

2 Gravitational instability

In our previous work [17], numerical methods investigated collisional relaxation. The distribution of particles in the space at the initial moment of time was set to be uniform. As is known, the initial uniform distribution of particles is unstable and, under the influence of gravitational forces acting between the particles, breaks up into separate clumps.

For the first time, Jeans formulated and solved the problem of the stability of a uniform distribution of matter [37, 38]. The appearance of celestial bodies and their systems occurs precisely due to the decay of the initial uniform distribution of particles that takes place at the initial stages of the cosmological evolution of the Universe [39-45]. The development of instability occurs due to the competition of two factors: gravity, which tends to collect matter into separate clumps, and pressure, which tends to equalize the resulting non-uniformities.

Let us briefly outline the derivation of the development of instability (see, for example, [40]). The equations of hydrodynamics and gravitation in the Newtonian approximation for an ideal gas have the form:

$$\frac{\partial \rho}{\partial t} + div(\rho\vec{u}) = 0,$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}grad)\vec{u} + \frac{grad(p)}{\rho} + grad(\phi) = 0,$$

$$\Delta\phi = div\big(grad(\phi)\big) = 4\pi G\rho,$$

$$\frac{\partial s}{\partial t} + (\vec{u}grad)s = 0,$$

where ρ is density, $\vec{u}$ is velocity, p is pressure, s is the specific entropy of matter, φ is the gravitational potential, G is the gravitational constant. When considering unstable modes of an undisturbed stationary gas uniformly distributed in space, perturbations are sought in the form of a plane wave superimposed on an undisturbed solution with a frequency ω and a wave vector $\vec{k}$:

$$z = z_0 + \delta(z)\, exp(\,\omega t + i\vec{k}\vec{x}), \tag{1}$$

where z_0 is the undisturbed value of the magnitude z. As an undisturbed state, we consider a gas at rest ($\vec{u} = 0$) uniformly distributed in space ($\rho = \rho_0$ = const, $S = S_0$ = const, $P = P(\rho_0, S_0)$ = const, grad $\varphi = 0$).

The resulting system of equations because of substituting the desired quantities of the form (1) has a nontrivial solution for

$$\omega = \pm\sqrt{4\pi G\rho - b^2k^2},$$

where $b^2 = \frac{\partial p}{\partial \rho}$. For long-wave perturbations, the time of increase of perturbations by e times is the value

$$\tau' = \frac{1}{\omega},$$

and in the limit $k \to 0$ ($\lambda \to \infty$) and $\omega \to \sqrt{4\pi G\rho_0}$ is

$$\tau_J = \frac{1}{\sqrt{4\pi G\rho_0}} = \frac{2}{\sqrt{3}\pi}\tau_f, \qquad (2)$$

where

$$\tau_f = \sqrt{\frac{3\pi}{16G\rho_0}}, \qquad (3)$$

is the free fall time of the point mass to the center of a homogeneous ball of density ρ_0.

Note that random perturbations of a uniform distribution lead to different consequences in the limit of short- and long-waves. In the long-wave limit ($\lambda > \lambda_J$)

$$\lambda_J = \frac{2\pi}{k_J} = b\sqrt{\frac{\pi}{G\rho_0}},$$

the uniformity decays in time τ`, approximately equal to τ_J. In the short-wave limit ($\lambda < \lambda_J$), perturbations lead to the excitation of waves against the background of this distribution.

3 Gravitationally coupled collisionless system

The decay of a uniform distribution of gravitating particles occurs not only due to gravitational instability. Uniform distribution in gravitational systems is also destroyed due to the mechanism of collisionless relaxation (see details in [46-50]). This mechanism is associated with the phase mixing of such a system. When particles move in a stationary potential field ($\frac{\partial\Phi}{\partial t} = 0$), the intrinsic energy of a particle does not depend on time and is equal to

$$\varepsilon = \frac{v^2}{2} + \Phi,$$

where Φ is the potential of the field, and ν is the velocity of the particle, and

$$\frac{d\varepsilon}{dt} = \frac{1}{2}\frac{dv^2}{dt} + \frac{d\Phi}{dt} = \vec{v}\left(\frac{d\vec{v}}{dt} + \nabla\Phi\right) + \frac{\partial\Phi}{\partial t} = 0.$$

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If the potential changes with time, then the energy of the particles also changes

$$\frac{d\varepsilon}{dt} = \frac{1}{2}\frac{dv^2}{dt} + \frac{d\Phi}{dt} = \vec{v}\left(\frac{d\vec{v}}{dt} + \nabla\Phi\right) + \frac{\partial\Phi}{\partial t} = \left.\frac{\partial\Phi}{\partial t}\right|_{\vec{x}(t)}.$$

The particles are redistributed in the phase space, and the change in their energy, unlike collisional relaxation, does not depend on the particle's mass. The time of such relaxation in the case of energy fluctuations can be estimated as [46-50]

$$\tau_{LB} \approx \left[\frac{1}{\varepsilon^2}\left(\frac{d\varepsilon}{dt}\right)^2\right]^{\frac{1}{2}}.$$

This time can be associated not with the rate of change in the particle energy but with the rate of change in the potential and the rate of change in the effective radius of the system. Let us consider the spherical collapse of a homogeneous ball of collisionless particles with a total mass of M. It follows from the virial theorem that $2\bar{W}_k \approx -\bar{U}$ or $\bar{U} \approx 2E$, where E is the total energy of the system, W_k is the total kinetic and U is the total potential energy of the system (the bar at the top means the average over time). In this case, given that$E = -\bar{W}_k$, we get $(1/2)mv^2 \approx -(1/2)m\Phi$ or $\varepsilon \approx (1/2)\Phi$, which gives

$$\tau_{LB} \approx \left[\frac{1}{\Phi^2}\left(\frac{d\Phi}{dt}\right)^2\right]^{\frac{1}{2}}.$$

The effective radius of the system of particles can be determined from the condition $U = -\frac{GM^2}{R}$. The angular momentum of system I is proportional to MR^2 with some constant ι^2:

$$I = \iota^2 MR^2.$$

Then, from the nonequilibrium virial theorem [50]

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2W_k + U,$$

we have

$$\iota^2 \left[R\frac{d^2R}{dt^2} + \left(\frac{dR}{dt}\right)^2 \right] = -\frac{2E}{M} - \frac{GM}{R} = \frac{2|E|}{M} - \frac{GM}{R}.$$

As a result of the occurrence of the δ$R(t)$ fluctuation, it is possible to write $R(t) = \bar{R} + \delta R(t)$ and, decomposing this differential equation for $R(t)$ near its equilibrium value

$$\bar{R} = -\frac{GM^2}{2E},$$

we obtain an equation for δ$R(t)$

$$\frac{d^2\delta R}{dt^2} = \frac{GM}{\iota^2 \bar{R}^3}\delta R,$$

the perturbation growth index of which is determined by the expression

$$\omega_{LB}^2 = \frac{GM}{\iota^2 \bar{R}^3} = \frac{4\pi G}{3\iota^2}\bar{\rho},$$

where $\bar{\rho} = \frac{3M}{4\pi \bar{R}^3}$ is the average density of the system. By relating the effective potential to the effective radius of the system

$$\Phi \approx -\frac{GM}{NR},$$

$$\frac{d\Phi}{dt} \approx -\frac{GM}{NR^2}\frac{dR}{dt},$$

where N is the number of particles in the system, we obtain

$$\tau_{LB} \approx \left[\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2\right]^{\frac{1}{2}} \approx \omega_{LB}^{-1} = \sqrt{3}\iota\tau_J = \frac{2\iota}{\pi}\tau_f. \quad (4)$$

That is, the time of collisionless relaxation coincides in order of magnitude with the time of development of gravitational instability (2) and, accordingly, in order of magnitude coincides with the time of free fall (3), i.e. the time of the collapse of a homogeneous cold extended mass into a point. If the initial distribution of particles inside the sphere is uniform, then

$$\iota^2 = \frac{2}{5},$$

and the time of collisionless relaxation differs from the time of development of gravitational instability by 10%.

4 Modelling and discussion of the results

Systems with long-range interaction, which include the gravitational and Coulomb interactions, are usually described by the ideality parameter $\delta = \langle U \rangle / \langle W_k \rangle$, (or sometimes by $\gamma \cong \delta^3$) representing the system's mean potential $\langle U \rangle$ energy ratio to its mean kinetic energy $\langle W_k \rangle$. This parameter characterizes the degree of the ideality of the system under consideration. In the ideal case, $\delta \ll 1$, the consideration of many issues related to the behaviour of such systems is usually greatly simplified.

To consider the instability, a series of calculations was carried out in non-ideal

$$\delta \gg 1,$$

and almost ideal

$$\delta \approx 1,$$

cases.

In connection with questions of substantiation of the statistical description, it was important to us to be convinced that the system we simulated has gravitational instability. The evolution of the spatial distribution of particles and questions connected to a range of system parameters, with which splitting the system into clumps of particles is possible, were not considered and analyzed.

***Case A* (δ ≈ 1).** The calculation parameters were as follows: number of particles was 300, particle mass was 10^{-9} kg, cube edge length was 10^{-6} m, and initial kinetic energy of all particles was the same $W_k = 0.05$ eV. At the initial moment of time, the particles in the cube and the directions of their velocities were distributed uniformly. The absolute values of the particle velocities were set to be the same at the initial moment of time, especially to trace the evolution (see Figs. 1, 2 in [17]) of the particle distribution function in terms of total and kinetic energies (or velocities). Under the conditions listed above, the total potential energy of the system of particles at the initial moment of time was $U = -35.7$ eV (this corresponds to the mean potential energy of <U> = –0.119 eV). Thus, the absolute value of the ideality parameter at the initial moment of time was $\delta \approx 2.4$, and the time for the development of gravitational instability (and collisionless relaxation) according to (2) and (4) should be 0.063 s. This value is in good agreement with the results of numerical calculations. Indeed, the initial distribution of particles begins to deform approximately from 10^{-2} s, and by 10^{-1} s concentrations for different groups of particles change by a factor of two or more times (Fig. 1). Recall that the distribution of particles in kinetic energy is almost completely formed only by 1-2 s; it has the form of a Maxwell distribution with a temperature approximately equal to 0.08 eV (see Figs. 1, 2 in [17]). At the initial moment of time, it is impossible to talk about the temperature of particles, since all particles have the same kinetic energy. It makes sense to talk about temperature only at the moment of relaxation time (about 2 s, see [17] for more details), at which the ideality parameter is already $\delta \approx 1.66$.

The relaxation time obtained in the numerical simulations [17] coincides well with the time obtained in the theoretical consideration of relaxation processes. However, the question arises: until when should the calculations be performed to ensure the relaxation process is over? It may turn out that the system will continue to evolve. For example, the relaxation time in numerical experiments can be significantly longer than τ, and then on time scales of the order of τ, the changes in the distribution function are simply almost imperceptible. In this case, the virial theorem comes to the rescue. In systems with gravitational interaction, the average absolute value of δ should be 2 if particles occupy a limited volume. The absolute value of parameter δ goes to the value of 2 by about 0.5 s and then does not change, i.e. it means that the system turns to the stationary state.

It seemed it is possible to object that nothing prevents to vary <U> and <W_k> so that the statement of the virial theorem [51] remains true

$$2 < W_k >= k < U >= -< U >, \quad (5)$$

(k = –1 for gravitational interaction). However, it not so. In view of that

$$< W_k > + < U >=< E >= E,$$

we have

$$< U >= \frac{2}{k+2} E = 2E,$$

$$< W_k >= \frac{k}{k+2} E = -E,$$

whence it is visible, that if <U> and <W_k> are in accordance with (5), they already further cannot vary.

Note also, that if we remove the walls in the calculations, then the particles continue to move in a compact region without escaping, despite the absence of any boundaries, and the absolute value of parameter δ goes to the value of 2 also by about 0.5 s and then does not change. Consequently, there will be no further redistribution of the values of the kinetic and potential energies in the system.

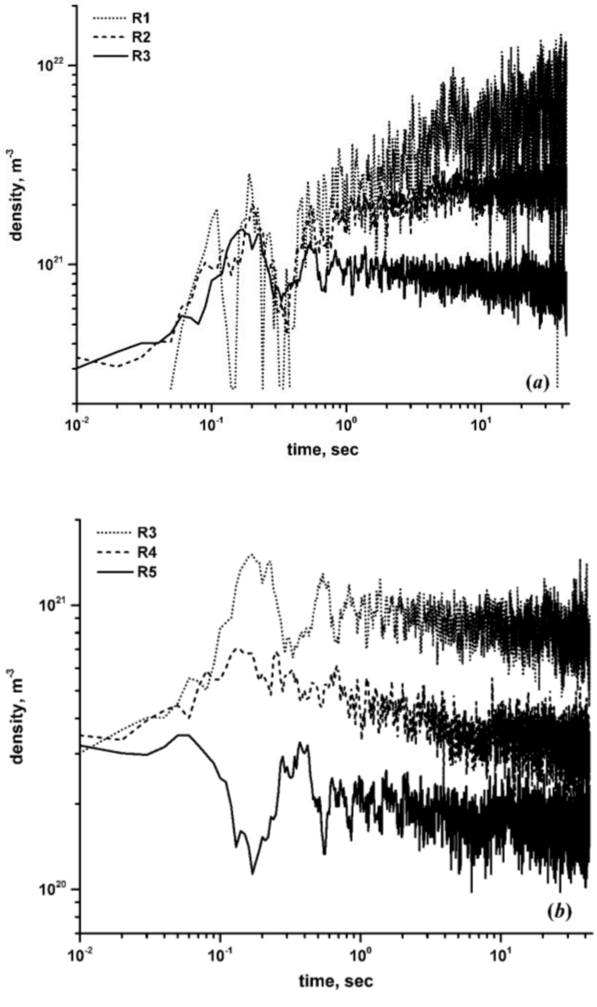

Fig. 1. Dependencies of particle density on time. The particles (300 particles each of mass 10^{-9} kg) are initially uniformly distributed in a cube with the edge of $a = 10^{-6}$ m. The designation on the graph R_i corresponds to the density of particles in a spherical layer from $0.5 \cdot a \cdot (i-1)/n$ to $0.5 \cdot a \cdot (i)/n$, where $n = 5$ is the total number of spatial partitioning layers and $i = 1, ..., n$ is the number of a specific layer.

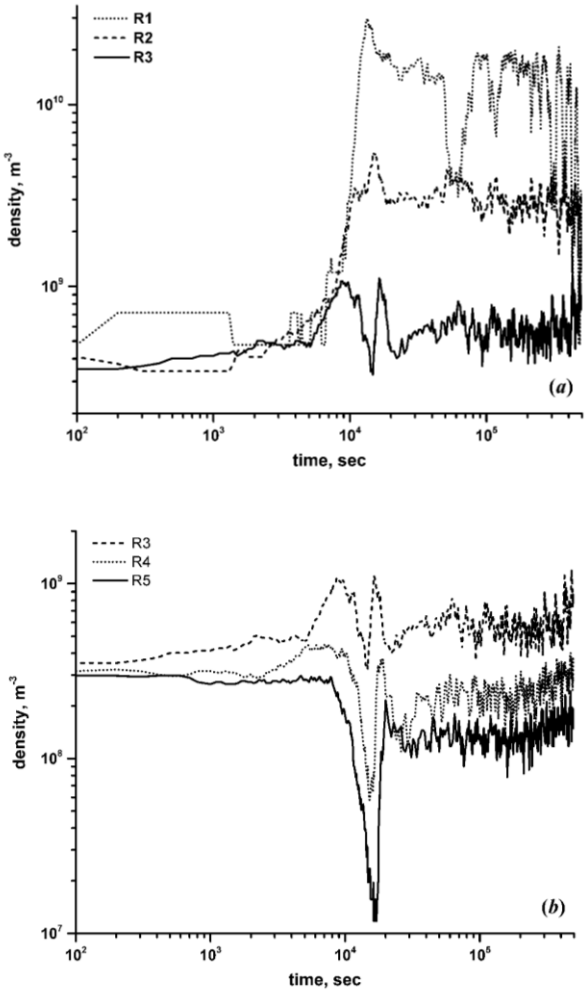

Fig. 2. Dependencies of particle density on time. The particles (300 particles each of mass 10^{-7} kg) are initially uniformly distributed in a cube with the edge of $a = 10^{-2}$ m. The designation on the graph R_i corresponds to the density of particles in a spherical layer from $0.5 \cdot a \cdot (i-1)/n$ to $0.5 \cdot a \cdot (i)/n$, where $n = 5$, $i = 1, ..., n$.

Case B **($\delta >> 1$).** The calculation parameters were as follows: number of particles was 300; particle mass was 10^{-7} kg; temperature of particles was $T = 10^{-2}$ eV (originally Maxwell distribution); cube edge length was 10^{-2} m. At the initial moment of time, the particles in the cube and the directions of their velocities were distributed uniformly. In this case, the absolute velocities of the particles at the initial moment of time were not set the same. However, there are no qualitatively significant differences in the evolution (see Figs. 3, 4 in [17]) of the particle distribution function in terms of total and kinetic energies (or velocities) compared to the previous case *A*. Under the above conditions, the total potential energy of the particle system at the initial moment of time was $U = -36.2$ eV, so the absolute value of the ideality parameter at the initial moment of time was $\delta \approx 8.04$ (if we take into account that $< W_k > = 1.5T$). The time for the development of gravitational instability (and collisionless relaxation), according to (2) and (4), should be $6.3 \cdot 10^3$ s. This value is in good agreement with the results of numerical calculations. Indeed, the initial distribution of particles begins to deform approximately starting from $4 \cdot 10^3$ s and by 10^4 s, concentrations for different groups of particles vary by a factor of two or more times (Fig. 2). Recall that the kinetic energy distribution of particles is almost completely formed by $1.5 \cdot 10^4$ - $2 \cdot 10^4$ s; it has the form of a Maxwellian distribution with a temperature approximately equal to 0.12 eV (see Figs. 3, 4 in [17]).

The relaxation time obtained in the numerical simulations [17] coincides well with the time obtained in the theoretical consideration of relaxation processes. Just as in the case *A*, the average absolute value of δ should be 2. Note, that if we remove (or not remove) the walls in the calculations, then this value goes to the value of 2 to about 25000 s and then does not change. Therefore, there will be no further redistribution of the values of the kinetic and potential energies in the system. Just as in the case *A*, (if walls were removed) the particles continue to move in a limited area without escaping.

5 Conclusions

Numerical simulation of the behaviour of classical particles interacting gravitationally with each other was carried out. The conducted modelling is directly related to the question of the possibility of substantiating the statistical behaviour of classical mechanical systems based on their mechanical behaviour. The cases with the ratio of the potential energy to the kinetic energy (ideality parameter) of $\delta \geq 1$ at the initial conditions were considered. The main results obtained in the work can be summarized as follows.

1. The decay of a homogeneous distribution of gravitationally interacting particles occurs due to the development of gravitational instability. The time of development of this instability is well known. It was obtained using the Jeans approach and in subsequent works by various authors. The collisionless relaxation mechanism of gravitational systems can also contribute to the collapsing of a homogeneous particle distribution. The duration of both mechanisms is approximately the same and coincides in magnitude with the time obtained in the numerical simulation carried out in this paper.

2. The relaxation time of the system obtained based on the numerical simulation [17] is also in good agreement with the relaxation time obtained in a large number of studies using various approximations.

3. However, the purely gravitational interaction of particles does not lead to the formation of Boltzmann form's particle energy distribution function in the region

of large magnitude negative energies [17]. Such behaviour must occur in accordance with the known theorems when the system approaches its equilibrium. Almost more than a century of attempts have been made to justify such a transition and to concordance the statistical and mechanical approaches in such a transition.

4. On the other hand, such behaviour should not also occur in accordance with other known theorems. In the language of entropy, the transition to the equilibrium corresponds to the system's transition to the state with the highest possible entropy. However, the well-known entropy conservation theorems for mechanical closed systems in both classical and quantum cases prevent such a transition since the entropy of such systems cannot change.

5. Thus, the current work demonstrates another example of a system in which, when approaching its equilibrium, the distribution function does not take the canonical form if the system does not involve any other factors besides its mechanical behaviour. Earlier, a similar result was demonstrated in the example of a classical Coulomb plasma.

6. Thus, it is generally impossible to justify the transition of the distribution function of a closed system in equilibrium to the canonical Gibbs distribution only based on its mechanical behaviour for systems with long-range Coulomb or gravitational interactions. For such a justification, it is necessary to consider the presence of other processes in the system in addition to only the mechanical interaction of particles with each other. For example, the system must have a stochastiser in one form or another, which will remove the ban on changing the entropy of closed systems imposed by the well-known entropy conservation theorems.

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