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Quantum acoustics: Investigation of phonons, their physical properties, and interactions with quantum systems

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Abstract Phonons, as quantum quasiparticles representing collective vibrations in crystal lattices, play a fundamental role in understanding the physical properties of solid materials. This article explores the physical foundations of phonons, including their types (acoustic and optical phonons), dispersion relations, and the influence of effective mass on their propagation. It further examines the interaction of phonons with quantum systems, such as their coupling to superconducting qubits, their role in quantum information transfer, and their application in quantum memories. Additionally, cutting-edge quantum acoustics technologies are discussed, including bulk acoustic wave (BAW) and surface acoustic wave (SAW) propagation, the piezoelectric effect in phonon generation and control, and phononic waveguides. The final section addresses technical challenges and future research directions in quantum acoustics, analyzing the potential of this field for advancing quantum information processing and emerging technologies.

1 Introduction

Quantum acoustics, as an emerging branch of modern quantum physics, focuses on studying the quantum behavior of mechanical waves and their interactions with quantum systems, opening new frontiers in quantum engineering and information technologies. This field leverages fundamental concepts of phonons (quanta of lattice vibrations in solids) to explore quantum phenomena such as entanglement, superposition, and quantum information transfer in acoustic systems [1]. Surface acoustic waves (SAWs) and bulk acoustic waves (BAWs) serve as key tools in this domain, enabling low-loss, controlled-speed transmission of quantum information (Fig. 1). Recent studies have demonstrated that these waves can bridge superconducting Josephson-junction qubits and semiconductor quantum dots, paving the way for hybrid quantum processing architectures [2].

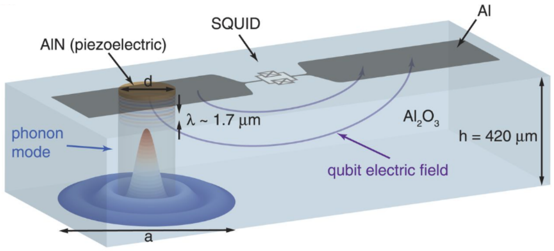

Fig. 1 Schematic of a piezoelectric coupling to modes of a high-overtone bulk acoustic resonator (HBAR) (not to scale). The longitudinal wavefunction component is depicted as a sinusoidal profile with wavelength $\lambda = 2h/l$ across the cylindrical mode volume defined by an AlN disk and a sapphire substrate. The transverse energy density profile $s_{l,0}(x/v)$ is rendered in 3D, illustrating effective energy confinement within the mode volume while accounting for diffraction-induced leakage. This also shows that the $s_{l,0}(x/v)$ mode is equivalent to the $s'_{l,3}(x/v)$ mode in a larger volume with diameter a [3].

A milestone in quantum acoustics was the achievement of coherent quantum state transfer between physically separated nodes, realized in 2022 using an advanced unidirectional piezoelectric transducer. These experiments employed single-phonon transmission over 2-mm channels with a propagation time of 500 ns, highlighting the scalability potential for quantum architectures. These advances relied on low-loss (<0.1 dB/cm) phononic waveguides with coherence times of $\sim 2.2\ \mu$s [4]. Meanwhile, recent theoretical studies on non-Markovian phonon-qubit dynamics have provided new insights into decoherence mechanisms and mitigation strategies. For instance, in lithium niobate systems, engineered phonon anisotropy reduced thermal noise by 40% at 10 mK [3].

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Technical challenges in this field primarily stem from surface phonon scattering and environmental factors like thermal fluctuations. Researchers have optimized hybrid piezo-optomechanical structures, achieving strong coupling ($>$1 MHz) between microwave phonons and optical photons, a key step toward quantum frequency converters. In 2024, a breakthrough experiment using phononic metamaterials demonstrated squeezed quantum states in acoustic waves, enabling ultra-sensitive quantum sensors [5].

Future directions include integrated quantum acoustic networks interfacing with optical and microwave processors. Early studies on time-periodic phononic crystals have shown dynamic control over phonon propagation via geometric modulation, potentially enabling all-acoustic quantum logic gates. Additionally, discoveries of collective quantum effects (e.g., Majorana-like quasiparticles in acoustic chains) are expanding topological quantum research.

Practically, quantum acoustic systems are increasingly used in quantum memories with storage times exceeding 10 ms. A recent innovation employed high-Q (10^6) acousto-optic cavities to store and retrieve single-photon states with 85% efficiency [6]. These advances, combined with phononic cooling techniques reaching sub-millikelvin temperatures, are accelerating the development of scalable quantum systems.

2 Foundations of Quantum Acoustics: Mathematical Analysis and Quantum Interactions

Quantum acoustics systematically studies the quantum behavior of mechanical waves in material media, where energy scales become sufficiently small that quantum effects such as superposition, entanglement, and projective measurements dominate the system's dynamics. The theoretical core of this field is based on the quantization of acoustic fields and non-classical interactions between phonons and other qubits. The classical wave equation in homogeneous elastic media is expressed as [7]:

$$\frac{\partial^2 \psi(\mathbf{r},t)}{\partial t^2} = v^2 \nabla^2 \psi(\mathbf{r},t), \tag{1}$$

where ψ represents particle displacement, v is the speed of sound, and ∇^2 is the Laplacian operator. For quantum description, this equation must be rewritten using secondary quantization methods. By introducing phonon creation ($b_k^\dagger$) and annihilation (b_k) operators, the displacement field is quantized as:

$$\hat{\psi}(\mathbf{r},t) = \sum_k \sqrt{\frac{\hbar}{2\rho V \omega_k}} \left(b_k e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)} + b_k^\dagger e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)} \right). \tag{2}$$

Here, ρ is the medium density, V is the system volume, $\omega_k = v|\mathbf{k}|$ is the acoustic phonon dispersion relation, and $\mathbf{k}$ is the wave vector. This formulation shows that phonons behave as massless quantum particles with energy $E_k = \hbar\omega_k$ and momentum $\mathbf{p} = \hbar\mathbf{k}$ [8].

At low temperatures (below 100 mK), phonons follow Bose-Einstein statistics, with density of states given by $g(\omega) = \frac{V\omega^2}{2\pi^2 v^3}$. Under such conditions, quantum effects like phononic squeezed states, which reduce quantum fluctuations in one phase component, become observable. These states are generated using squeeze operators $S(\zeta) = \exp\left(\frac{1}{2}\zeta^* b^2 - \frac{1}{2}\zeta (b^\dagger)^2\right)$, where $\zeta = re^{i\theta}$ is the squeezing parameter.

Photon-phonon interactions in piezoelectric materials are modeled using quantum elasticity theory and Maxwell's equations. In these materials, mechanical stress (σ_{ij}) and electric field (E_k) are coupled through piezoelectric coefficients (e_{ijk}) [9]:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \tag{3}$$

$$D_i = e_{ikl}\varepsilon_{kl} + \varepsilon_{ij}E_j, \tag{4}$$

where c_{ijkl} is the elasticity tensor, ε_{kl} the strain tensor, and ε_{ij} the permittivity tensor. At the quantum level, these interactions are described by an extended Jaynes-Cummings Hamiltonian:

$$\begin{aligned} H = {} & \hbar\omega_c a^\dagger a + \hbar\Omega_m b^\dagger b \\ & + \hbar g_0 a^\dagger a (b + b^\dagger) + \hbar J (a^\dagger b + a b^\dagger). \end{aligned} \tag{5}$$

The first two terms represent cavity photons (frequency ω_c) and mechanical phonons (frequency Ω_m), respectively. The third term (linear electromechanical coupling, strength g_0) and fourth term (nonlinear coupling, strength J) describe energy transfer between the systems. Recent experiments on lithium niobate platforms measured $g_0/2\pi \sim 10-100$ kHz and $J/2\pi \sim 1-10$ kHz, demonstrating their strong potential for quantum processing.

A key challenge in quantum acoustics is managing decoherence from phonon-environment interactions. Surface acoustic wave (SAW) phonon decoherence rates are estimated by:

$$\Gamma_\phi = \frac{k_B T \omega_m^2}{Q_m \hbar \Omega_m}, \tag{6}$$

where Q_m is mechanical quality factor, T temperature, and Ω_m mechanical frequency. Below 20 mK with $Q_m > 10^6$, decoherence rates fall below 100 Hz, enabling coherence times of ∼10 ms.

Quantum information transfer via phonons requires a low-loss waveguide design. Phonon propagation in acoustic waveguides follows:

$$\frac{\partial \hat{b}(x,t)}{\partial t} = -v_g \frac{\partial \hat{b}(x,t)}{\partial x} - \frac{\kappa}{2}\hat{b}(x,t) + \sqrt{\kappa}\hat{b}_{in}(t), \tag{7}$$

where v_g is phonon group velocity, κ loss rate, and $\hat{b}_{in}$ the input operator. For $> 90\%$ transfer efficiency, the condition $\kappa L/v_g \ll 1$ must hold, recently achieved in gallium arsenide (GaAs) waveguides with $L = 2$ mm and $\kappa/2\pi = 50$ Hz.

Nonlinear effects in quantum acoustics, like phonon-phonon interactions, are described by [10]:

$$H_{nonlinear} = \hbar\chi(b^\dagger)^2 b^2, \tag{8}$$

where χ is the nonlinear coefficient. These effects are observable in highly anisotropic materials (e.g., diamond), enabling phononic squeezed states and multiphonon entanglement. In 2023, silicon nanoparticle experiments reported nonlinear interactions with $\chi/2\pi \sim 1$ kHz, paving the way for all-acoustic quantum gates.

3 Phonons and Their Properties

Phonons are fundamental quantum mechanical concepts in solid-state physics that describe collective atomic vibrations in crystalline lattices. When atoms in a solid are at their equilibrium positions, the crystal lattice is stable. However, mechanical disturbances propagate as waves through the material. While macroscopically these are sound waves, at microscopic quantum scales, they are quantized waves called phonons. Essentially, phonons are the mechanical analogs of photons in electromagnetism and play vital roles in materials' thermodynamic and electrical properties [7].

A key aspect of phonons is their influence on thermal conductivity. In insulators where heat transfer occurs primarily through phonons, understanding their dynamics enables precise descriptions of thermal behavior. Phonon-electron interactions also critically determine electrical and thermoelectric properties. For instance, in superconductors, phonon-electron coupling creates the superconducting phase where electrical resistance vanishes.

In crystalline structures, atoms are connected by electrostatic forces. When displaced, this motion propagates through interatomic forces as vibrational waves throughout the lattice. These waves, influenced by the material's mechanical and crystalline properties, form different phonon types.

Phonons are categorized as either **acoustic** or **optical**. Acoustic phonons exist in all solids and involve in-phase atomic oscillations, propagating near the material's sound speed. They facilitate mechanical and thermal energy transfer and are subdivided into longitudinal (atomic motion parallel to wave propagation) and transverse (perpendicular motion) modes [11].

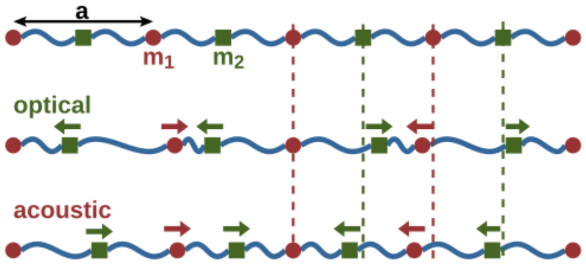

Fig. 2 Schematic of crystal lattice showing acoustic and optical phonon propagation

Optical phonons primarily occur in crystals with multiple atom types per unit cell, where atoms oscillate out of phase. This enables strong interactions with electromagnetic waves, particularly in infrared and ultraviolet regions, significantly impacting optical properties through phenomena like Raman scattering and infrared absorption. Like acoustic phonons, optical phonons have longitudinal and transverse variants (Fig. 2).

The phonon dispersion relation – describing how frequency (ω) depends on wavenumber (k) – is crucial for understanding phonon behavior. This relation is nearly linear for acoustic phonons at long wavelengths ($omega \propto k$), resembling sound waves in continuous media. At higher frequencies, crystal symmetries and interatomic interactions introduce nonlinearities.

Unlike acoustic phonons, optical phonons typically exhibit nonlinear dispersion, maintaining nearly constant frequencies at small wavenumbers because their vibrational energy depends on unit cell structural changes rather than wavenumber. Consequently, optical phonons have higher frequencies (often several THz) than acoustic phonons (Fig. 3).

Atomic mass significantly influences phonon properties. In heavier elements, higher effective phonon masses reduce wave propagation speeds, directly lowering thermal conductivity since slower phonons transfer less heat (Fig. 4).

Phonon interactions with other quantum particles are research hotspots. Phonon-photon interactions occur in Raman scattering and photoelastic effects, while phonon-electron coupling enables phenomena like electron-phonon scattering and superconductivity. In quantum systems, phonons can mediate qubit coupling for quantum information transfer.

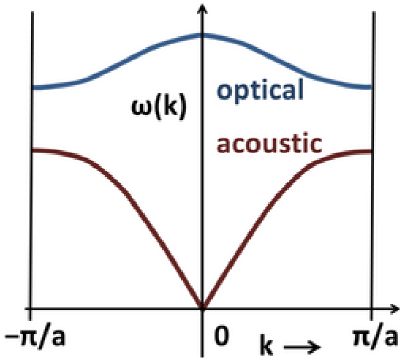

Fig. 3 Dispersion curves for acoustic and optical phonons in a linear diatomic chain

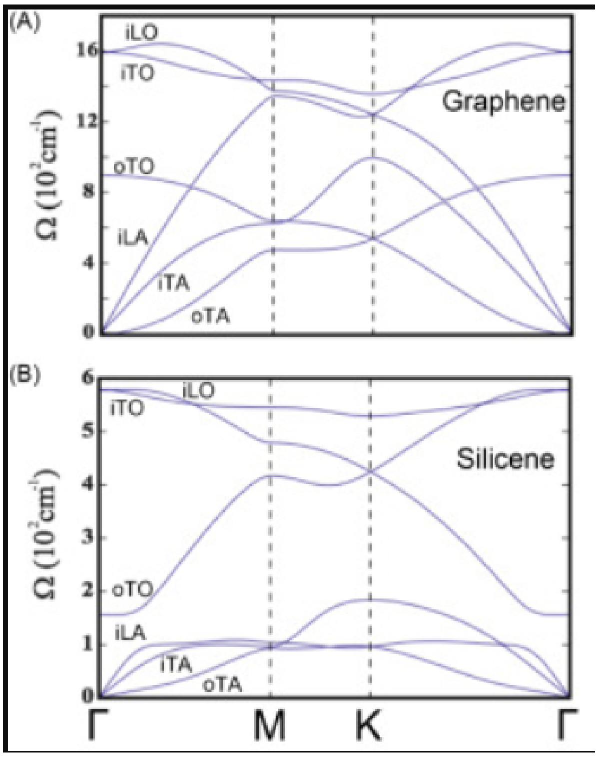

Fig. 4 Vibrational properties of silicene and graphene: (A-B) Calculated phonon dispersion with labeled modes (i: in-plane, o: out-of-plane, A: acoustic, O: optical, L: longitudinal, T: transverse) [4]

Overall, phonons are essential for understanding solid-state physics across thermodynamics, optics, electronics, and quantum computing. Recent research on phononic engineering and controlled phonon guidance structures highlights their potential for developing quantum information technologies and ultra-sensitive mechanical sensors [8, 12].

4 Interaction of Phonons with Quantum Systems

Phonons play a crucial role as quantum information carriers in the development of quantum processing and quantum communications. Unlike photons that propagate in vacuum, phonons are dependent on material media, yet this characteristic makes them particularly suitable for certain specialized applications. In some systems, phonons can serve as efficient mediators for quantum information transfer due to their long coherence times and low loss. In systems based on superconducting qubits and quantum dots, surface acoustic waves (SAWs) and bulk acoustic waves (BAWs) have been proposed for establishing connections between different components of a quantum circuit. The propagation of single-mode phonons in a controlled environment enables coherent quantum information transfer, a feature of paramount importance for establishing quantum networks [9, 11].

In quantum information processing, phonons can be engineered through various approaches. One of their most significant capabilities in this domain is their use in creating single-mode phononic states. In this method, individual phonons are generated and controlled in specific media, enabling quantum information storage and processing. In certain materials, phonons can preserve information for extended periods, making them excellent candidates for constructing quantum memories. Phononic quantum memories can serve as intermediaries for information exchange between different sections of a quantum processor, ensuring the prolonged stability of quantum information.

One of the most important interactions of phonons in quantum physics is their coupling with superconducting qubits (Fig. 5). Superconducting qubits are typically controlled through electromagnetic fields, but their interaction with phonons can also be achieved via the piezoelectric effect. In this interaction, phonon energy can induce qubit state transitions, and conversely, qubits can generate or absorb phonons. This coupling is described by various theoretical models, including the Jaynes-Cummings Hamiltonian:

$$H = \hbar\omega_q \sigma^+ \sigma^- + \hbar\omega_p a^\dagger a + \hbar g(\sigma^+ a + \sigma^- a^\dagger), \tag{9}$$

where ω_q is the qubit frequency, ω_p the phonon frequency, $a^\dagger$ (a) are phonon creation (annihilation) operators, $\sigma^\pm$ are qubit raising/lowering operators, and g is the coupling strength [8, 13].

This model demonstrates that a superconducting qubit can change states by absorbing or emitting a phonon. Such phenomena provide a foundation for developing phonon-based quantum circuits where quantum information is stored and processed in qubits and phonons. The interaction between phonons and superconducting qubits also plays a role in developing quantum sensors. Due to phonons' sensitivity to environmental changes, they can be used to detect extremely weak mechanical fields, potentially leading to the development of ultra-precise sensors for measuring faint forces and structural changes in materials.

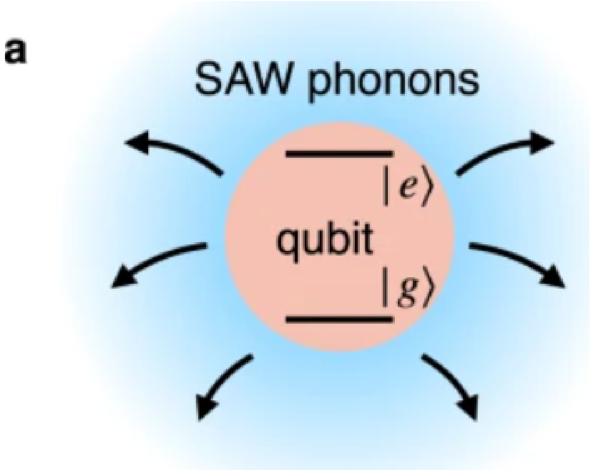

Fig. 5 Schematic diagram illustrating the coupling of a qubit to a SAW phonon bath: The qubit (orange-pink) non-uniquely emits excitations to the SAW phonon bath (blue), with the emission rate regulated by the SAW structure's electrical conductivity [14].

In summary, phonons offer unique capabilities for quantum information processing and transfer. While photons remain the primary mediators for quantum communications, phonons – with their stronger interaction with solid-state environments and ability to couple with mechanical and electronic systems – represent an exceptionally suitable platform for advancing quantum technologies. The integration of these characteristics with emerging phonon control and engineering techniques may open new pathways in quantum computing and sensing [15].

5 Quantum Acoustics-Based Technologies

Quantum acoustics, as an interdisciplinary field combining quantum mechanics and acoustic physics, encompasses technologies that utilize quantized sound waves (phonons) for information transfer and processing. These technologies are primarily categorized into three main areas: acoustic wave propagation, phonon control and generation via the piezoelectric effect, and phononic waveguides.

Acoustic waves propagating in solid materials are broadly classified into bulk acoustic waves (BAW) and surface acoustic waves (SAW). In BAW, acoustic vibrations propagate throughout the material volume and can be guided along specific paths depending on boundary conditions and material structure. Due to their deep penetration, BAWs are commonly used in high-precision sensors and quantum communication systems. In contrast, SAWs propagate only along surface layers and remain concentrated near the surface. This property gives SAWs high sensitivity to surface changes, making them suitable for frequency filters, precision sensors, and as quantum system interconnects in quantum communications [7, 8, 13].

One of the most important methods for phonon control and generation in quantum acoustics is the piezoelectric effect (Fig. 6). Observed in materials like aluminum nitride, quartz, and certain ceramics, this effect generates an electric field under mechanical stress and vice versa - applying an electric field induces mechanical deformation. This property enables precise phonon control and acoustic wave generation in solids. For example, applying alternating voltage to a piezoelectric material produces acoustic waves at specific frequencies that can guide, store, or transfer quantum information. This technology plays a key role in manufacturing quantum chips, requiring precise phononic wave control.

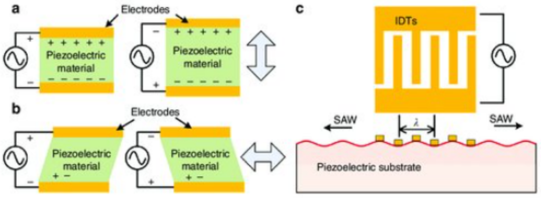

Fig. 6 Acoustic wave generation via piezoelectric materials. a) Applying voltage to electrodes causes thickness-mode vibrations (perpendicular expansion/contraction). b) For certain material orientations, applied voltage induces shear-mode vibrations (horizontal deformation). c) Interdigital transducers (IDTs) on piezoelectric crystals generate surface acoustic waves (SAWs) with wavelength (λ) determined by IDT finger spacing [16].

Phononic waveguides represent another crucial quantum acoustics technology enabling controlled phonon propagation (Fig. 7). Similar to optical waveguides in photonics, specially designed structures can guide phononic waves along predetermined paths. These waveguides are typically created by locally modifying mechanical properties like density or stiffness. Their key advantage is enabling phonon path control without external fields - essential for designing phonon-based quantum circuits [7, 17].

6 Practical Applications of Quantum Acoustics

With advancements in quantum acoustics technologies, the use of phonons for quantum information transfer has become one of the most important applications in this field (Fig. 7). While many quantum systems use photons for information transfer, phonons have emerged as a superior alternative for certain systems due to their lower environmental sensitivity and more precise controllability. In some systems, quantum qubits based on electronic states can interact with phonons, enabling information transfer between different components of a quantum processor. This capability is

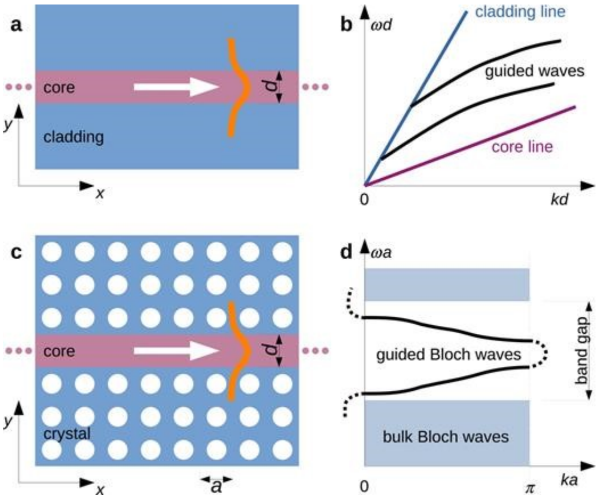

Fig. 7 Waveguide principles. a) Homogeneous waveguides contain a core and cladding. When the wave velocity is lower in the core, waves propagate in the core while being evanescent in the cladding. b) Dispersion of guided waves between two sound lines (core and cladding). c) In phononic crystal waveguides, the cladding is replaced by a complete bandgap phononic crystal. d) Being periodic along its axis, the phononic crystal waveguide exhibits folded dispersion bands within bandgaps, defined for Bloch wavenumbers in the first Brillouin zone [18].

particularly valuable for technologies requiring communication between superconducting qubits.

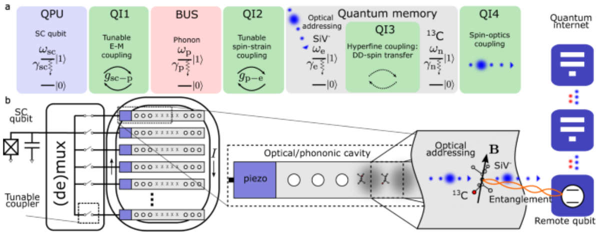

Fig. 8 (a) A superconducting quantum processing unit (QPU) connects to a phononic bus via piezoelectric "Quantum Interface 1" (QI1). The phonon couples with electron spin-orbit states of an active center (AA), forming "Quantum Interface 2" (QI2). The active center's microstructure states can further couple with nuclear spins via "Quantum Interface 3" (QI3) to form a quantum memory (QM), or connect to photons through "Quantum Interface 4" (QI4), ultimately linking to the quantum internet (blue dots: optical connections). (b) Implementation shows a superconducting qubit connected via phononic/microwave multiplexers to waveguide arrays, each interacting with mechanical cavities hosting active centers (AA). These centers' electron spin-orbit states serve as qubits, hyperfine-coupled to adjacent carbon-13 nuclear spins for high-coherence auxiliary qubits. Optical transitions provide quantum network interfaces through electron-photon spin entanglement protocols. This architecture utilizes: tunable electromechanical qubit-cavity coupling, tunable spin-strain coupling, optical spin addressing, and hyperfine coupling for nuclear quantum memory [19].

Another emerging application is phonon-based quantum memory. A major challenge in quantum computing is maintaining quantum coherence over extended periods. Phonons, with their weaker environmental interactions compared to photons and other quantum information carriers, can serve as quantum memory in certain systems. These memories utilize controlled mechanical oscillations created by phonons in materials, enabling longer-duration quantum information storage [1, 13, 15].

Quantum acoustics also enables the detection of acoustic waves at quantum levels. Phonon-sensitive systems, often based on superconducting qubits or other quantum sensors, can detect extremely weak mechanical vibrations. This has important applications in seismology, material defect detection, and even gravitational wave research [14, 17].

7 Future Prospects and Challenges

Quantum acoustics, while possessing transformative potential, faces several hurdles that must be addressed to realize its full capabilities. A primary challenge lies in the inherent difficulty of achieving precise control over phonons. Unlike photons, which propagate freely through a vacuum, phonons are intrinsically tied to the material media through which they travel. This dependence on the medium complicates their manipulation and control, adding a layer of complexity not present in photonic systems. Furthermore, the delicate nature of phonons makes them susceptible to environmental noise. Unexpected interactions with the surrounding environment can induce phonon decoherence, a process that leads to the degradation and eventual loss of the quantum information encoded within the phonons. This decoherence poses a significant obstacle to the development of robust quantum acoustic devices.

Another significant challenge stems from the experimental limitations that currently constrain our ability to generate and detect single phonons with high fidelity. While substantial progress has been made in recent years, achieving complete and reliable control over individual phonons remains an ongoing pursuit. Overcoming this limitation requires the development and refinement of advanced technologies, including improved piezoelectric devices capable of efficiently converting electrical signals into acoustic waves, sophisticated nanostructures designed to guide and manipulate phonons at the nanoscale, and highly sensitive quantum sensors capable of detecting individual phonons with minimal disturbance. Continued advancements in these areas are crucial for unlocking the full potential of quantum acoustics.

Despite these existing challenges, quantum acoustics offers immense promise for revolutionizing the field of quantum information processing. By leveraging the unique properties of phonons, researchers hope to develop more stable and efficient quantum information systems that surpass the limitations of current technologies. The true potential of quantum acoustics is likely to be realized through its synergistic integration with other quantum technologies, creating hybrid quantum systems that combine the strengths of different physical platforms. Future progress in the field will likely be driven by focused research efforts aimed at elucidating the intricate interactions between phonons and other quantum components, designing and fabricating integrated acoustic circuits capable of performing complex quantum operations, and developing improved phononic sensors with enhanced sensitivity and precision [4, 9, 10, 12, 14].

The ongoing research efforts in quantum acoustics, particularly when considered within the broader context of other domains of quantum physics, hold the potential to yield groundbreaking discoveries that could transform various aspects of our lives. These advancements may lead to breakthroughs in quantum communications, enabling secure and ultra-fast data transmission; quantum computing, paving the way for solving currently intractable computational problems; and ultra-sensitive sensing, allowing for the detection of minute changes in the environment with unprecedented accuracy. Ultimately, the continued exploration and development of quantum acoustics may unlock entirely new pathways for scientific and technological advancement, leading to innovations that were previously unimaginable [11, 20–22].

As part of our future research endeavors, we plan to dedicate our efforts to investigating the seamless integration of quantum acoustic systems with photoacoustic imaging [3, 14, 16, 17, 22, 23] platforms. Our primary focus will be on enhancing the overall image quality achievable through this integration, with a particular emphasis on leveraging phonon-based quantum sensing techniques to improve sensitivity and resolution. Furthermore, we will explore the application of sophisticated phononic circuit engineering principles to optimize the performance of the integrated imaging system, aiming to develop a powerful and versatile tool for a wide range of applications.

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