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Effect of time reversal symmetry on the current-phase relation in Josephson junctions including two-gap superconductors

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Abstract From a general viewpoint, the Josephson Effect is a quantum phenomenon that the tunnelling of Cooper electron pairs can describe. This junction involves two or more superconductors separated by a thin layer of non-superconducting material. A supercurrent can flow without dissipation through this junction without applying voltage. The dependence of the Josephson current on the phase difference between the two superconductors is of significant importance and is investigated in this study. This research considers a Josephson junction consisting of two gap superconductors separated by a thin insulator. Additionally, we consider a spin-singlet s-wave pair potential in each conduction band. We study the effect of time-reversal symmetry on the phase difference and, consequently, on the Josephson current in two cases. The first case is when only one of the superconductors has time-reversal symmetry, and the second is when neither superconductor has time-reversal symmetry. These states are compared with the condition where both superconductors have time-reversal symmetry.

1 Introduction

In 1962, Josephson predicted that a supercurrent could exist in a junction consisting of two superconductors separated by a thin insulator (typically about 1 nm) [1]. This phenomenon was experimentally confirmed by Shapiro [3–5]. Research on Josephson junctions remains essential due to their wide applications, including SQUIDs, electrical metrology, and digital memory circuits [4, 6, 7]. The initial structure of the Josephson junction was the Superconductor-Insulator-Superconductor (SIS) junction. Over the decades, alternative structures such as Superconductor-Ferromagnetic-Superconductor (SFS) [6–9] and Superconductor-Insulator-Ferromagnetic-Superconductor (SIFS) [6–10] have been investigated. Various aspects of Josephson junctions have been studied, including their current-voltage (I-V) characteristics, temperature dependence [11], frequency dependence [12], electrodynamics, and the Current-Phase Relation (CPR) [2]. Some studies on Josephson junctions' current-voltage (I-V) characteristics have shown that the significant nonlinearity in the I-V characteristics makes them suitable for digital and pulsed devices [13]. Nevertheless, the Current-Phase Relation (CPR) is one of the fundamental properties of a Josephson junction. When an insulator separates two superconductors, the Josephson current is typically proportional to $\sin\theta$, where $\theta = \varphi_R - \varphi_L$ denotes the phase difference between the right (φ_R) and left (φ_L) superconductors [14]. If the superconductors have two or more conduction bands, as in iron pnictides, the relation becomes more complex [15]. Studies show that even in the absence of a phase difference between the superconductors, a Josephson current can arise due to the internal phase difference between bands [15–18]. While the BCS theory adequately describes conventional superconductors, many unconventional superconductors deviate from BCS predictions. Time-reversal symmetry is generally preserved in the mean-field Hamiltonian of two-band superconductors [19]. However, mounting evidence suggests that time-reversal symmetry breaking (TRSB) occurs in several unconventional systems [20–22]. The time-reversal operation affects the Hamiltonian H such that $\Theta H \Theta^{-1} = H$. TRSB occurs when this condition is violated [19, 23, 24]. Besides intrinsic magnetism, local disruptions can also break time-reversal symmetry, influencing the local quasiparticle spectrum. Vortices carrying fractional flux become strongly pinned to domain walls in superconductors with broken time-reversal symmetry, leading to atypical flux-flow dynamics [24].

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This research considers a junction between two dissimilar double-band s-wave superconductors (Figure 1). After deriving the current-phase relation, we examined the dependence of the Josephson current on the phase under two conditions. In the first case, we assumed that time-reversal symmetry was preserved in one of the superconductors but broken in the other. The results showed that the current-phase relationship is exactly the opposite of the case where time-reversal symmetry is preserved in both superconductors. In the second case, we assumed that time-reversal symmetry was broken in both superconductors. Under these conditions, the results were similar to the case where both superconductors were in a time-reversal symmetric state.

In contrast to previous studies, such as Sasaki et al. [15], which focused on time-reversal symmetric Josephson junctions composed of two-band superconductors, our work investigates both symmetric and time-reversal symmetry-breaking configurations. In particular, we explore the case in which time-reversal symmetry is broken in only one of the superconductors. This asymmetric situation leads to non-trivial modifications in the current-phase relation, including spontaneous supercurrents that appear even without phase difference. Furthermore, our analysis includes the role of inter-band hybridization in such time-reversal symmetry-broken systems, revealing new contributions to the Josephson current that were not previously addressed.

2 Introduction to Current-Phase Relation

When an electron tunnels through a barrier, it generates an electron state on one side and a hole state on the opposite side (left and right) of the barrier. The Hamiltonian describing this process can be expressed as [2, 15, 18]:

$$H_J = H_L + H_R + \mathscr{T}, \tag{1}$$

$$\mathscr{T} = \sum_{p,q,\sigma} \left(\mathscr{T}_{pq} a^{\dagger}_{\sigma p} a_{\sigma q} + \mathscr{T}^{*}_{pq} a^{\dagger}_{\sigma q} a_{\sigma p} \right), \tag{2}$$

where H_L and H_R denote the Hamiltonians of the left and right superconductors, respectively, and $\mathscr{T}_{pq}$ describes the tunneling matrix elements that determines the probability of tunneling. The operators $a^{\dagger}_{p\sigma}$ and $a_{p\sigma}$ are the creation and annihilation operators for electrons on the left superconductor with momentum $\mathbf{p}$ and spin σ, respectively. Similarly, $a^{\dagger}_{q\sigma}$ and $a_{q\sigma}$ are the corresponding operators for electrons on the right superconductor with momentum $\mathbf{q}$ and spin σ.

The Josephson current can be expressed as [25]:

$$J = 2e\,\mathrm{Im}\left[T \sum_{i\omega_n} \sum_{k,p} \mathrm{Tr}\left(\hat{t}_T\, \hat{\mathscr{F}}^{\dagger}_{R}(k,i\omega_n)\, \hat{t}_T\, \hat{\mathscr{F}}^{\dagger}_{L}(k,i\omega_n) \right) \right], \tag{3}$$

where $\hat{\mathscr{F}}_L(k,i\omega_n)$ and $\hat{\mathscr{F}}_R(k,i\omega_n)$ are the anomalous Green's functions for the left and right superconductors, respectively. Here, $\hat{\mathscr{F}}^{\dagger}_{R}(k,i\omega_n) = \hat{\mathscr{F}}_R(-k,-i\omega_n)$. The operator Tr denotes the trace over internal degrees of freedom (e.g., spin or band indices), T is the temperature, e is the electron charge, and Im denotes the imaginary part. The matrix $\hat{t}_T$ represents the tunneling matrix between the superconductors.

The fermionic Matsubara frequencies are given by $\omega_n = (2n+1)\pi T, \quad n = 1,2,3,\ldots$ The current-phase relation (CPR) is a central feature of the Josephson junction. In only a few special cases, it simplifies to the classical sinusoidal form with a critical current J_c [18], namely:

$$J = J_c \sin\varphi, \tag{4}$$

The maximum current J_c in the current-phase relation (CPR) is called the *critical current*. In general, it depends on the temperature, the magnetic field, and may include effects due to fluctuations [2]. Additionally, the phase difference of the superconducting order parameters across the junction is given by $\varphi = \varphi_R - \varphi_L$, where φ_R and φ_L are the phases of the right and left superconductors, respectively [15].

In general, the dependence of the supercurrent J on the phase difference φ can be written as a Fourier series [2]:

$$J(\varphi) = \sum_{n\geq 1} \left[J_c \sin(n\varphi) + J_n \cos(n\varphi) \right], \tag{5}$$

It is predicted that if time-reversal symmetry is preserved, the cosine harmonic coefficients J_n in the Fourier expansion vanish [2, 19, 26, 27]. However, some characteristics of the current-phase relation (CPR) are quite universal and do not depend on the specific material, geometry of the junction, or theoretical framework used to describe it. For example, reversing the direction of supercurrent flow necessarily results in a reversal of the phase difference's sign [5]:

$$J(\varphi) = -J(-\varphi). \tag{6}$$

Moreover, $J(\varphi)$ is a 2π-periodic function [5], i.e.,

$$J(\varphi) = J(\varphi + 2\pi). \tag{7}$$

The investigation of the CPR in diverse junctions, such as superconductor-insulator-superconductor (SIS), double-barrier, superconductor-ferromagnet-superconductor (SFS), and superconductor-constriction-superconductor point-contact junctions, shows that the CPR exhibits different structures for each type of junction [16, 17, 19].

3 Description of Our Model

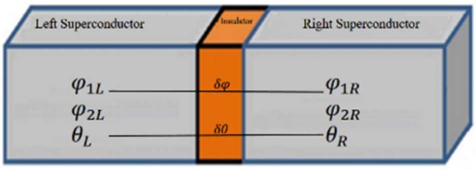

Fig. 1 Schematic picture of the Josephson structure considered in this article.

The supercurrent consists of three distinct terms in a Josephson junction consisting of two dissimilar s-wave spin-singlet two-gap superconductors separated by an insulator. The first term arises from the tunneling of electron pairs between the first conduction band, the second term arises from the tunneling of electron pairs between the second conduction band, and the third term corresponds to the tunneling of electron pairs induced by band hybridization.

In the following equations, the currents resulting from the tunneling of electron pairs in the first and second conduction bands are denoted by J_1 and J_2, respectively. The current arising from the tunneling of an induced electron pair due to band hybridization is given by J_{12} [15, 16]. In these expressions, the indices $\alpha = L, R$ refer to the left and right superconductors, and the band indices $\beta = 1, 2$ represent the first and second conduction bands. The quantity υ_α denotes the hybridization amplitude between the two bands in superconductor α, while $\xi_{\alpha\beta}$ is the kinetic energy of an electron in band β on side α. The superconducting order parameter is denoted by $\Delta_{\alpha\beta}$, and T represents the temperature. The tunneling matrix elements through the barrier in band β on side α are represented by t_α. Finally, the phase factors are given by $e^{i\varphi_{\alpha\beta}}$ and $e^{i\theta_\alpha}$, where $\varphi_{\alpha\beta}$ is the phase of the superconducting order parameter in band β of superconductor α, and θ_α is an internal phase related to time-reversal symmetry.

3.1 First Condition: Just One Superconductor is in a Time-Reversal Symmetry State

$$
\begin{aligned}
J_1 = \mathrm{Im}\sum_{kp} eT \sum_{\omega_n} \Bigg[& \frac{t_1^2 \left[\left(\xi_{2R}^2 + |\Delta_{2R}|^2 + \omega_n^2\right) |\Delta_{1R}| + \upsilon_R^2 |\Delta_{2R}|\, e^{i(2\theta_R + \varphi_{2R} - \varphi_{1R})} \right]}{\left(\xi_{1R}^2 + |\Delta_{1R}|^2 + \omega_n^2\right) \left(\xi_{2R}^2 + |\Delta_{2R}|^2 + \omega_n^2\right) + \upsilon_R^2 \left(\omega_n^2 - \xi_{1R}\xi_{2R}|\Delta_{1R}||\Delta_{2R}| \cos(\varphi_{2R} - \varphi_{1R} + 2\theta_R) + \upsilon_R^4\right)} \\
& \times \frac{\left[\left(\xi_{2L}^2 + |\Delta_{2L}|^2 + \omega_n^2\right) |\Delta_{1L}| + \upsilon_L^2 |\Delta_{2L}|\, e^{i(2\theta_L + \varphi_{1L} - \varphi_{1L})} \right] \cdot e^{i(\varphi_{1L} - \varphi_{1R})}}{\left(\xi_{1L}^2 + |\Delta_{1L}|^2 + \omega_n^2\right) \left(\xi_{2L}^2 + |\Delta_{2R}|^2 + \omega_n^2\right) + \upsilon_L^2 \left(\omega_n^2 - \xi_{1L}\xi_{2L}|\Delta_{1L}||\Delta_{2L}| \cos(\varphi_{2L} - \varphi_{1L} + 2\theta_L) + \upsilon_L^4\right)} \Bigg],
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
J_2 = \mathrm{Im}\sum_{kp} eT \sum_{\omega_n} \Bigg[& \frac{t_2^2 \left[\left(\xi_{1R}^2 + |\Delta_{1R}|^2 + \omega_n^2\right) |\Delta_{2R}| + \upsilon_R^2 |\Delta_{1R}|\, e^{i(2\theta_R + \varphi_{2R} - \varphi_{1R})} \right]}{\left(\xi_{1R}^2 + |\Delta_{1R}|^2 + \omega_n^2\right) \left(\xi_{2R}^2 + |\Delta_{2R}|^2 + \omega_n^2\right) + \upsilon_R^2 \left(\omega_n^2 - \xi_{1R}\xi_{2R}|\Delta_{1R}||\Delta_{2R}| \cos(\varphi_{2R} - \varphi_{1R} + 2\theta_R) + \upsilon_R^4\right)} \\
& \times \frac{\left[\left(\xi_{1L}^2 + |\Delta_{1L}|^2 + \omega_n^2\right) |\Delta_{2L}| + \upsilon_L^2 |\Delta_{1L}|\, e^{i(2\theta_L + \varphi_{2L} - \varphi_{1L})} \right] \cdot e^{i(\varphi_{2L} - \varphi_{2R})}}{\left(\xi_{1L}^2 + |\Delta_{1L}|^2 + \omega_n^2\right) \left(\xi_{2L}^2 + |\Delta_{2R}|^2 + \omega_n^2\right) + \upsilon_L^2 \left(\omega_n^2 - \xi_{1L}\xi_{2L}|\Delta_{1L}||\Delta_{2L}| \cos(\varphi_{2L} - \varphi_{1L} + 2\theta_L) + \upsilon_L^4\right)} \Bigg],
\end{aligned}
\tag{9}
$$

$$
J_{12} = \mathrm{Im}\sum_{kp} eT \sum_{\omega_n} \left[\frac{\begin{gathered} t_1 t_2 \Big(\big[\xi_{1R}|\Delta_{2R}|e^{i(\varphi_{2R} - \theta_R} + \xi_{2R}|1R|e^{i(\varphi_{1R} - \theta_R} \big] \big[\big(\xi_{1L}|\Delta_{2L}|e^{i(\varphi_{2L} - \theta_L} \big) + \xi_{2L}|\Delta_{1L}|e^{i(\varphi_{1L} - \theta_L} \big] \\ + \big[|\Delta_{2R}|e^{i(\varphi_{2R} - \theta_R)} - |\Delta_{1R}|e^{i(\varphi_{1R} - \theta_R)} \big] \big[|\Delta_{2L}|e^{i(\varphi_{2L} - \theta_L} - |\Delta_{1L}|e^{i(\varphi_{1L} - \theta_L} \big] \omega_n^2 \Big) \upsilon_R \upsilon_L \cdot e^{i\theta_R}\, e^{-i\varphi_{1R}} \end{gathered}}{\begin{gathered} \Big\{ \big(\xi_{1R}^2 + |\Delta_{1R}|^2 + \omega_n^2 \big) \big(\xi_{2R}^2 + |\Delta_{2R}|^2 + \omega_n^2 \big) + \upsilon_R^2 \big(\omega_n^2 - \xi_{1R}\xi_{2R}|\Delta_{1R}||\Delta_{2R}| \cos(\varphi_{2R} - \varphi_{1R} + 2\theta_R) + \upsilon_R^4 \big) \Big\} \\ \times \Big\{ \big(\xi_{1L}^2 + |\Delta_{1L}|^2 + \omega_n^2 \big) \big(\xi_{2L}^2 + |\Delta_{2R}|^2 + \omega_n^2 \big) + \upsilon_L^2 \big(\omega_n^2 - \xi_{1L}\xi_{2L}|\Delta_{1L}||\Delta_{2L}| \cos(\varphi_{2L} - \varphi_{1L} + 2\theta_L) + \upsilon_L^4 \big) \Big\} \end{gathered}} \right] \tag{10}
$$

To observe the time-reversal symmetry state, the phases of the first and second conduction bands must satisfy:

$$2\theta_R = \varphi_{1R} - \varphi_{2R} + 2\pi n_R. \tag{11}$$

Therefore, for the state in which time-reversal symmetry is broken [16]:

$$2\theta_R \neq \varphi_{1R} - \varphi_{2R} + 2\pi n_R. \tag{12}$$

So, the relationship between the phases in the state of broken time-reversal symmetry could be assumed as

$$2\theta_R = \varphi_{1R} - \varphi_{2R} + \pi n_R. \tag{13}$$

Based on the phase configuration assumed in Eq. (13), and considering the three distinct tunneling contributions in Eqs. (8)–(10), we derive the general form of the current-phase relation (CPR) when time-reversal symmetry is broken in only one superconductor:

$$\begin{aligned}
J &= J_1' \sin(\delta\varphi) + J_2' \sin(-2\delta\theta + \delta\varphi) + J_{12}' \sin(\delta\varphi - \delta\theta) \\
\Longrightarrow J &= J' \sin(\delta\varphi) \\
&\quad + J_2'[\sin(\delta\varphi)\cos(2\delta\theta) - \sin(2\delta\theta)\cos(\delta\varphi)] \\
&\quad + J_{12}'[\sin(\delta\varphi)\cos(\delta\theta) - \sin(\delta\theta)\cos(\delta\varphi)],
\end{aligned} \tag{14}$$

where $J'_{1(2,12)}$ represent the amplitudes of the currents corresponding to the first, second, and third components, respectively, and are temperature-dependent. We define the relative phase difference between the superconductors as $\delta\theta = \theta_L - \theta_R$, which depends on the intrinsic characteristics of the superconductors. Additionally, we consider $\delta\varphi_1 \equiv \varphi_{1L} - \varphi_{1R}$, and define the average phase as$\varphi_{1L(R)} = \frac{1}{2}\left(\varphi_{1L(R)} + \varphi_{2L(R)}\right)$. Regarding the internal phase relations within each superconductor, two possible configurations are considered. In the S++ configuration, the phases satisfy $\varphi_1 = \varphi_2$, while in the S+− configuration, the relative phase satisfies $\varphi_1 - \varphi_2 = \pi$.

3.1.1 S++/S++ Junction

In this situation, for the superconductor that preserves time-reversal symmetry, we have $\delta\theta = -\frac{\pi}{2}, \frac{\pi}{2}$ while for the other superconductor, $\theta_L = \frac{\pi}{2}, -\frac{\pi}{2}$. Considering Eq. (14), the current-phase relation (CPR) takes the form:

$$\delta\theta = \pm\frac{\pi}{2} \Longrightarrow \\ J = J_1' \sin(\delta\varphi) - J_2' \sin(\delta\varphi) \mp J_{12}' \cos(\delta\varphi), \tag{15}$$

3.1.2 S+-/S+- Junction

Similar to the previous situation, in this junction configuration we also obtained $\delta\theta = \frac{\pi}{2}, -\frac{\pi}{2}$, and the result is consistent with the findings in Section 3.1.1.

3.1.3 S++/S+- Junction

Under this condition, we consider the phase relation $\varphi_{1R} - \varphi_{2R} = \pi$, which implies $\theta_R = \frac{\pi}{2}, -\frac{\pi}{2}, \quad \theta_L = \frac{\pi}{2}, -\frac{\pi}{2}$, and thus, $\delta\theta = 0$, π, or $-\pi$. Based on Eq. (14), for this junction configuration, the sign reversal in one of the bands leads to partial cancellation in the hybridized current contributions modifying the overalln (CPR) with an inherent asymmetry not present in fully symmetric configurations. The interference terms may no longer add constructively, giving rise to spontaneous currents or non-sinusoidal CPR asymmetry.

$$\delta\theta = 0 \Longrightarrow \\ J = J_1' \sin(\delta\varphi) + J_2' \sin(\delta\varphi) + J_{12}' \sin(\delta\varphi), \tag{16}$$

$$\delta\theta = \pm\pi \Longrightarrow \\ J = J_1' \sin(\delta\varphi) - J_2' \sin(\delta\varphi) - J_{12}' \cos(\delta\varphi), \tag{17}$$

It is obvious that θ affects the Josephson current in all the situations discussed above by modifying the sign of the last term in the current expression. Moreover, in $S{+}{+}/S{+}{+}$ and $S{+}{-}/S{+}{-}$ junctions, this last term leads to supercurrent flow even in the absence of a phase difference (i.e., $\delta\varphi = 0$). This behavior is in stark contrast to the behavior observed in Josephson junctions composed of two superconductors that both preserve time-reversal symmetry.

3.2 Second Condition: Both Superconductors Break Time-Reversal Symmetry

In this section, we consider Eq. (12) to be valid for both superconductors. All other assumptions remain the same as those outlined in Section 3.1.

3.2.1 S++/S++ Junction

Under this circumstance, for both superconductors, $\varphi_1 - \varphi_2 = 0$, and $\theta_{L(R)} = \frac{\pi}{2}, -\frac{\pi}{2}$. Therefore, $\delta\theta = 0, \pi$, and we conclude:

$$\delta\theta = 0 \Longrightarrow \\ J = J_1' \sin(\delta\varphi) + J_2' \sin(\delta\varphi) + J_{12}' \sin(\delta\varphi), \tag{18}$$

$$\delta\theta = \pi \implies$$
$$J = J'_1 \sin(\delta\varphi) + J'_2 \sin(\delta\varphi) - J'_{12} \cos(\delta\varphi), \quad (19)$$

3.2.2 S+-/S+- Junction

Similar to Section 3.2.1, in this junction the CPR is described by Eqs. (18) and (19).

3.2.3 S++/S+- Junction

In this section, we assumed $\theta_L = \pm\frac{\pi}{2}$ and $\theta_R = 0$. The CPR is described by:

$$\theta_L = \pm\frac{\pi}{2}, \theta_R = 0 \implies$$
$$J = J_1 \sin(\delta\varphi) - J_2 \sin(\delta\varphi) + J_{12} \pm \cos(\delta\varphi), \quad (20)$$

$$\theta_L = \pm\frac{\pi}{2}, \theta_R = \pi \implies$$
$$J = J_1 \sin(\delta\varphi) - J_2 \sin(\delta\varphi) + J_{12} \mp \cos(\delta\varphi), \quad (21)$$

Again, it is evident that θ modifies the Josephson current by altering the sign of the last term. However, this junction's result is similar to that of a junction consisting of two superconductors with time-reversal symmetry [15].

It is worth emphasizing that the sign structure of the superconducting gaps—i.e., whether the system is in an S++ or S± state—plays a crucial role in determining the behavior of the Josephson current. In S++ junctions, the constructive phase alignment between bands enhances the magnitude and symmetry of the CPR. In contrast, S± configurations introduce sign changes that may cancel or invert certain contributions, especially under broken time-reversal symmetry conditions.

We explicitly compare the S++ and S± configurations to clarify the qualitative impact of gap symmetry on the CPR. In the S++ state, both superconducting gaps have the same sign, leading to constructive interference between the interband tunneling contributions. As a result, the CPR is symmetric and typically dominated by first harmonic components.

However, in the S± state, the sign reversal between the bands introduces destructive interference, particularly in the current's interband (hybridization-induced) component. This can lead to significant deviations from the conventional sinusoidal CPR, including suppression of the critical current and the emergence of higher harmonics or phase shifts.

Figure 2 shows the CPR curves for a symmetric S++/S++ junction and a sign-reversed S±/S± junction, both under time-reversal symmetric conditions. As shown, the S± configuration leads to an overall reduction in current amplitude and a distortion in the CPR shape due to phase cancellation effects. This qualitative difference becomes even more pronounced when time-reversal symmetry is broken in one or both superconductors, leading to spontaneous currents and asymmetric CPRs, as discussed in previous sections.

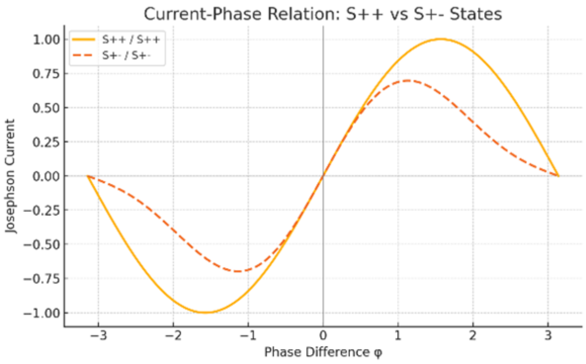

Fig. 2 Comparative CPR for S++ and S± states under time-reversal symmetric conditions.

4 Conclusions

A Josephson junction consisting of two dissimilar double-band superconductors separated by an insulating barrier was considered. The effect of time-reversal symmetry on the current-phase relation (CPR) was studied under two different conditions: first, when only one of the superconductors preserved time-reversal symmetry, and second, when both superconductors broke time-reversal symmetry. The results were compared with the case where both superconductors preserved time-reversal symmetry.

Contrary to an isolated superconductor, we conclude that the gauge parameter modifies the Josephson current. However, the CPR was similar when both superconductors were in the same time-reversal symmetry state—regardless of whether they broke or preserved the symmetry. In contrast, if only one broke the symmetry, the result was the opposite.

The modifications in the CPR observed under time-reversal symmetry breaking conditions can be intuitively understood as arising from the asymmetry in the phase structure of the superconducting order parameters. When time-reversal symmetry is broken, especially asymmetrically across the junction, complex inter-band phase differences and hybridization terms contribute to the Josephson current in a way that is not invariant under phase inversion. As a result, higher-order harmonics and phase shifts emerge in the CPR. In such scenarios, spontaneous supercurrents may appear even in the absence of an applied phase difference, highlighting the role of symmetry in shaping the transport properties of Josephson junctions.

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