TTMP vol 2 (3), 2025
doi: 10.30511/TTMP.2025.2065547.1058

Transactions in Theoretical and Mathematical Physics

Comparison of Proton Structure Functions in Classical and Quantum Viewpoints

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Received: 11 July 2025 / Accepted: 18 August 2025 / Published: 18 August 2025

Abstract A comparison between quantum statistical parton distributions and the classical perspective in the proton is presented. The structure functions and parton distributions are only provided by the entropy maximization principle and proton sum rules. In both the classical and quantum approaches, the equations for the number of proton valence quarks, conservation of momentum, and the first and second laws of thermodynamics are satisfied. However, if we use the Maxwell-Boltzmann distribution function to describe the partons in the proton structure function, the results show a clear deviation from the experimental results, while the Fermi-Dirac and Bose-Einstein distributions are in good agreement with the measurements from accelerators.

1 Introduction

The main goals of Deep Inelastic Scattering (DIS) of leptons on hadrons were to elucidate the internal hadron structure in terms of parton distributions. In the absence of a theory for the parton distributions experimentally, the distributions are approximated by different polynomials, which require numerous meaningless parameters [1, 2]. Cleymans and Thews [3], as pioneers, explored a statistical way to generate compatible Parton Distribution Functions (PDFs). Afterwards, the statistical viewpoint was applied by the other research groups [4–6]. According to the parton model, the proton is composed of a number of point-like constituents, named partons (quarks, anti-quarks and gluons). In the impulse approximation, the deep inelastic lepton–proton scattering can be viewed as a sum of elastic lepton–parton scattering, in which the incident leptons are scattered off partons instantaneously and incoherently. In the statistical approach, the proton is assumed to be a thermal system in equilibrium made up of free partons. The impulse approximation fails in the nucleon Rest Frame (RF), but works well in the Infinite Momentum Frame (IMF) [7]. In this paper, instant-form statistical expressions in the nucleon rest frame are transformed in terms of light-front kinematic variables. The longitudinal classical and quantum statistical parton distribution functions are expressed as a function of temperature, accessible volume and chemical potential for quarks, antiquarks, and gluons [8]. Antiparticles are primarily a Quantum Field Theory (QFT) construct, arising from the Dirac equation and the interpretation of negative energy solutions. In statistical mechanics or quantum thermodynamics, if antiparticles are considered in a grand canonical ensemble, their chemical potential could be taken as negative relative to their corresponding particles. The chemical potential is a measure of the change in free energy with a change in the number of particles. In classical physics, the idea of an anti-particle does not exist intrinsically. Antiparticles are a quan-tum concept with negative chemical potential due to the fact that antiparticles and particles can annihilate each other and be converted into energy. However, in modeling the classi-cal systems with antiparticles (e.g., particles with opposite charges), as a mathematical analogy, we consider them to have a negative chemical potential. The parameters are specified just by the proton sum rules and the momentum entropy optimization principle in both cases of classical and quantum statistics. Eventually, we investigate the agreement of the nucleon structure functions and parton distributions with the corresponding experimental results, based on quantum statistics compared to the classical case.

2 Theoretical Analysis

A nucleon is assumed to be a thermal system in equilibrium composed of quarks, antiquarks, and gluons in equilibrium, at a given temperature in a definite size. In the quan-

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tum ap-proach, quarks and antiquarks obey the Fermi-Dirac (FD) statistics.

$$q_Q = \frac{1}{e^{\left(\frac{1}{2}Mx-\mu\right)/T}+1}, \quad q_{\bar{Q}} = \frac{1}{e^{\left(\frac{1}{2}Mx+\mu\right)/T}+1}, \tag{1}$$

where M is the nucleon mass, μ is the chemical potential, and for the antiquark $\mu_{\bar{q}} = -\mu_q$. The transverse dis-tribution of partons is neglected as a reasonable approxima-tion without decreasing the worthiness of the framework. It is due to the independence of parton motion along and perpendicular to the nucleon axis in momentum space [9–11]. Concerning the gluon, the Bose-Einstein (BE) expres-sion is as follows:

$$g_Q = \frac{1}{e^{\left(\frac{1}{2}Mx\right)/T}-1}, \tag{2}$$

with a zero potential originated from gluon emission of fermionic partons at an intermediate value of Q^2due to evolution in the quantum chromodynamic (QCD) perturbative regime. It is in consistent with the fact that hadrons behave as black body cavities for the chromodynamical radiation [12]. On the other hand, in the classic approach, parton distributions are expressed by Maxwell Boltzmann (MB) statistic:

$$q_C = e^{-\left(\frac{1}{2}Mx-\mu\right)/T}, \tag{3a}$$

$$\bar{q}_C = e^{-\left(\frac{1}{2}Mx+\mu\right)/T}, \tag{3b}$$

$$g_C = e^{-\left(\frac{1}{2}Mx\right)/T}. \tag{3c}$$

Therefore, the momentum entropy of the partons inside a nucleon, in the momentum space for the both cases QSD and CSD are given by:

$$\begin{aligned} S_Q = &-\sum_{q,\bar{q}} [xq(x)\ln q(x) + (1-xq(x))\ln(1-xq(x))] \\ &- [g(x)\ln g(x) - (1-g(x))\ln(1-g(x))], \end{aligned} \tag{4}$$

$$S_C = -\sum_{q,\bar{q},g} [xf(x)\ln(xf(x)) - xf(x)]. \tag{5}$$

The momentum distribution is used for the sake of the fact that the structure function measured by experimentalists involves momentum distributions. Furthermore, the momentum distribution is applied to introduce the concept of parton that refers to QCD. The goal is to obtain the statistical variables T, V, μ in the proton for both the quantum and classical cases. In the proposed approach, thermodynamic parameters determining statistical parton distributions of the proton are given by momentum entropy maximization under proton sum rule conditions. In fact, the proton satisfies the entropy optimum principle:

$$\frac{MV}{2(2\pi)^3}\int_0^1 S(x)\,dx \longrightarrow \text{Max}, \tag{6}$$

with proton sum rule constraints, simultaneously:

$$\frac{MV}{2(2\pi)^3}\int_0^1 [u(x)-\bar{u}(x)]\,dx = 2, \tag{7a}$$

$$\frac{MV}{2(2\pi)^3}\int_0^1 \left[d(x)-\bar{d}(x)\right]\,dx = 1, \tag{7b}$$

$$\frac{MV}{2(2\pi)^3}\int_0^1 x\left[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)+g(x)\right]\,dx = 1. \tag{7c}$$

It is worth noting that in the expressed model, all parameters are statistical quantities. There is no arbitrary variable fixed by hand or fitted with the experimental data, which weakens the stringency of the approach.

3 Results and Discussion

In computing parameters, the mass of the proton, $M = 938\,\text{MeV}$, is taken as given and partons are assumed massless. Evaluated numerical values of statistical variables based on the maximum entropy method in the quantum case are $T_Q = 51.5\,\text{MeV}$, $V_Q = 0.60\,\text{MeV}^{-1}$, $\mu_u^Q = 136\,\text{MeV}$, $\mu_d^Q = 68\,\text{MeV}$.

On the other hand, for the classical case, computed statistical variables are $T_C = 97.6\,\text{MeV}$, $V_C = 1.88\,\text{MeV}^{-1}$, $\mu_u^C = 108\,\text{MeV}$, $\mu_d^C = 62\,\text{MeV}$. The composition of proton structure functions as a function of $\bar{u}$ and $\bar{d}$ is given by:

The composition of proton structure functions as a function of $\bar{u}$ and $\bar{d}$ is given by

$$F_2^p(x) = \tfrac{4}{9}\left(u(x)+\bar{u}(x)\right) + \tfrac{1}{9}\left(d(x)+\bar{d}(x)\right). \tag{8}$$

Whereas $F_2^n(x)$ is achieved from isospin symmetry approximation of the corresponding proton structure function:

$$F_2^n(x) = \frac{1}{9}\left(u(x) + \bar{u}(x)\right) + \frac{4}{9}\left(d(x) + \bar{d}(x)\right). \quad (9)$$

It is notable that the asymptotic behavior of the structure function ratio $F_2^n(x)/F_2^p(x)$ tends to $\frac{3}{7}$ as $x \to 1$, from the precise analysis of experimental data in favor of the perturbative QCD [13]. As illustrated in Fig. 1, the quantum statistical distribution (QSD) is in good accordance with the experimental data, rather than the classical statistical distribution (CSD).

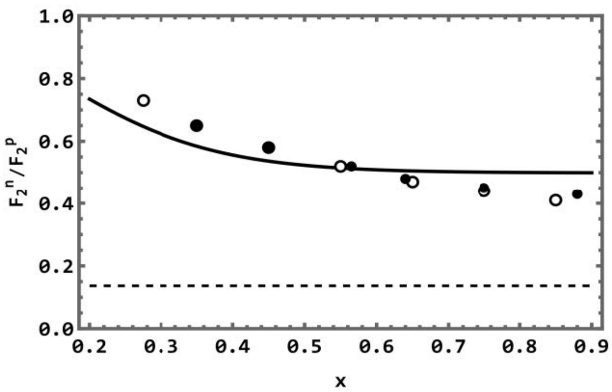
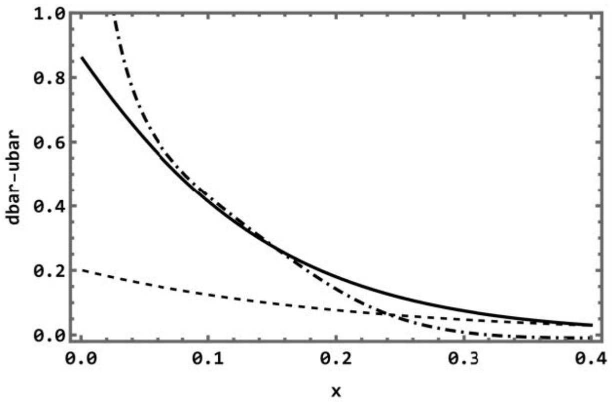
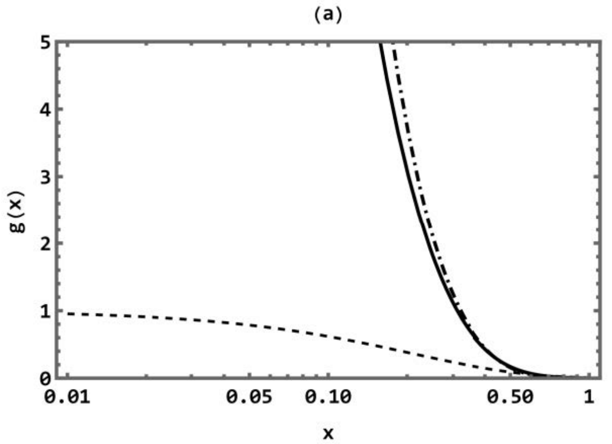


Fig. 1 Top: Comparison of our result for F_2^n/F_2^p (solid line QSD and dashed line CSD) with MST (hollow circles) and SLAC (filled circles) [13]. Bottom: Distribution of $\bar{d} - \bar{u}$ in our model (solid line QSD and dashed line CSD) compared to CTEQ6M (dash-dot line) [14].



(a)



(b)

Fig. 2 Distribution of $g(x)$ and $xg(x)$ in our model (solid line QSD and dashed line CSD) in comparison to Ref. [15] (dash-dot line).

In addition in Fig.1 Right, QSD shows a better agreement with accelerator observations which is originated from the nature of its statistic. The gluon distribution and its momen-tum distribution for the both cases of BE and MB distribu-tions are exhibited in Fig. 2. (a) and (b), respectively.

Comparison of the gluon distribution functions with the ex-perimental data in Fig. 2 shows a better agreement of the quantum gluonic distribution with the results of experiments. This fact stems from the bosonic nature of gluons. As it is clear from the obtained results, the QSDs give a fairly good description of structure functions of proton and its constituent parton distributions.

4 Conclusions

The partonic distribution functions describing the proton structure must obey quantum statistics to match experimental data.

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