



Metastable de Sitter Vacua from Critical Scalar Theory

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Abstract Studying the critical scalar theory in four dimensional Euclidean space with the potential term $-g\phi^4$ we show that the theory can not be analytically continued through $g = 0$ from $g < 0$ region to $g > 0$ region. For $g > 0$ although energy is not bounded from below but there exist a classical trajectory with an AdS_5 moduli space, corresponding to a metastable local minima of the action. The fluctuation around this solution is governed by a minimally coupled scalar theory on four dimensional de Sitter background with a reversed Mexican hat potential. Since in the weak coupling limit, the partition function picks up contribution only around classical solutions, one can assume that our de Sitter universe corresponds to that local minima which lifetime increases exponentially as the coupling constant tends to zero. Similar results is obtained in the case of critical scalar theory coupled to $U(1)$ gauge field which is essential for people living on flat Euclidean space to observe a de Sitter background by optical instruments.

Keywords: critical scalar theory; de Sitter vacua

1 Introduction

Recently, we showed that the fluctuations of the scalar field around the classical trajectory of massless ϕ^4 model in four dimensional flat Euclidean spacetime is governed by a conformally coupled scalar field theory in four dimensional de Sitter background [1]. This result is interesting due to its uniqueness. In four dimensions, in principle, one can consider two classes of critical (classically scale-free) scalar field theories i.e. massless ϕ^4 models on Euclidean (Minkowski) spacetime with g , the coupling constant, either positive or negative (we assume the potential $V(\phi) = -\frac{g}{4}\phi^4$). Although scalar theory with $g > 0$ seems

to be not physical as energy is not bounded from below but as is shown in [1] in this case, the Euler-Lagrange equation of motion of the scalar theory on Euclidean space has an interesting classical solution say ϕ_0 with finite action $S[\phi_0] \sim g^{-1}$. Interestingly, as far as we are considering real scalar field theories the model with the physical potential i.e. the case of $g < 0$ has not such a solution, see Eq.(1). We should clarify that from our previous viewpoint [1], for $g < 0$, one can still consider a solution like ϕ_0 obtained by an analytic continuation from $g > 0$ to $g < 0$ region. But such a solution is singular on the surface of a sphere which radius is proportional to g . Consequently the action $S[\phi_0]$ is infinite and ϕ_0 can not be considered as a classical trajectory. If one ignores this problem and follows the calculations one obtains a conformally coupled scalar field on AdS_4 background. The reason why we do not follow our previous point of view turns back to our new machinery for constructing ϕ_0 from the first principles explained in section 2. Of course similar singularities appear even for $g > 0$, when one switches to Minkowski spacetime by Wick rotation $x^0 \rightarrow ix^0$. But in this case ($g > 0$) one can argue that the singularity is beyond the horizon of observers living in the corresponding de Sitter spacetime and consequently is safe. We do not study the case of Minkowski spacetime in this paper.

As is shown in [1] the information geometry of the moduli of ϕ_0 is AdS_5 (dS_5) if $g > 0$ ($g < 0$) which resembles the information geometry of $k = 1$ $SU(2)$ instantons [2]¹. The moduli here are the location of the center of ϕ_0 and its size and are consequences of the invariance of the action under rescaling and translation. The similarity between this solution and $SU(2)$ instantons can be explained in terms of the 't Hooft ϕ^4 ansatz for instantons [3].

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¹ $V(\phi_0)$ can be shown to be proportional to the $SU(2)$ one-instanton density [2].

In this paper focusing on the case $g > 0$ we show that ϕ_0 can be responsible for viewing a metastable de Sitter background. We first show that ϕ_0 is a metastable local minima of the action. Then we show that the weak coupling limit $g \rightarrow +0$ is equivalent to the classical ($\hbar \rightarrow 0$) limit therefore in that limit the partition function picks up contribution only around the classical trajectory ϕ_0 . As we said before, fluctuations around ϕ_0 is governed by a scalar theory on a de Sitter background with a reversed Mexican potential. The height and width of the barrier is proportional to R and g^{-1} . Here R is the scalar curvature of the universe which can not be determined in our model but one can show that the lifetime of this local minima is proportional to $e^{g^{-1}}$.

Summarizing these results we verify that the critical scalar theory can not be analytically continued (at least through $g = 0$) from $g < 0$ region to $g > 0$ region. These results are strictly important from perturbation theory point of view. In the $g \rightarrow -0$ limit, valid perturbations are around $\phi = 0$ in a *flat* Euclidean background but in the $g \rightarrow +0$ limit perturbations are around $\phi = 0$ in an Euclidean *de Sitter* background. More explicitly at $g = 0^-$, the theory is simply a free massless scalar theory on flat Euclidean space but at $g = 0^+$ due to \hbar considerations the theory is a (free and stable) conformally coupled scalar theory on an Euclidean de Sitter background. See sections 2 and 3.

But why we are interested in de Sitter background. In fact WMAP results [4] combined with earlier cosmological observations shows that we are living in an accelerating universe. Constructing four dimensional de Sitter vacuum as a string theory (M-theory) solution has been a long standing challenge. In [5] KKLT constructed a metastable de Sitter vacua of type IIB string theory by adding $\overline{D3}$ -branes to the GKP [6] model of highly warped IIB compactifications with nontrivial NS and RR three-form fluxes. The mean lifetime of KKLT solution is $10^{10^{20}}$ years. KKLMNT constructed a model of inflation by adding mobile D3 branes to the KKLT solution [7]. In such models the inflaton (the position of D3 brane deep inside the warped throat geometry) is a conformally coupled scalar in the effective four-dimensional geometry. Therefore the mass of such a scalar m_ϕ^2 is close to $2H^2 = \xi R$ where H^2 is the Hubble parameter, $\xi = \frac{1}{6}$ (in four dimension) is the conformal coupling constant and R is the curvature of de Sitter space. As is uncovered in [7] this does not meet the observational requirement $m_\phi^2 \sim 10^{-2}H^2$.² Similar considerations show that our model can not be a successful model for inflation since in the $g \rightarrow +0$ limit we also obtain a conformally coupled scalar theory on de Sitter background.

²In D3/D7 model of inflation one does not encounter the $m_\phi^2 \sim H^2$ problem. But here in contrast to the model of D3/ $\overline{D3}$ inflation in the highly warped throat, there is no natural mechanism for suppressing the contribution of cosmic strings formed at the end of inflation to the CMB anisotropy. See [8] for a solution to this problem.

Another problem in our model is the existence of a continuum of de Sitter bubbles, given by the location of the center of ϕ_0 and its size. Naively this number is proportional to the volume of the AdS₅ moduli space. If ϕ_0 is responsible to observe, say, by *optical* methods a de Sitter geometry (see section 4), a natural question is to ask which bubble we live in. Studying the variation of action around the ϕ_0 solution, we have verified, by numerical calculations that smaller bubbles are more stable than larger ones. Consequently there is a transition: larger bubbles decay to smaller ones and probably finally there remains a gas of zero size bubbles. The mechanism of such a decays is not clear to us yet but its phenomenology, might be similar to that of the discretuum of possible de Sitter vacua in KKLT models [9].

As we said before our theory can not do any prediction about the size of our de Sitter universe or the nature of the scalar field but it predicts that the lifetime of this metastable vacua is $\tau \sim e^{g^{-1}}$. The model is interesting due to its uniqueness and symmetries which brings hopes to be constructible from a fundamental theory like string theory.

The organization of the paper is as follows. In section 2, we study the critical scalar theory in $D = 4$ Euclidean space and ϕ_0 , the exact solution to the corresponding Euler-Lagrange equation of motion. Considering the weak coupling limit we show that the partition function only picks up contribution around ϕ_0 . In section 3, we show that the critical scalar theory considered as the action for (not essentially small) fluctuations around ϕ_0 is a scalar theory on $D = 4$ de Sitter background. Consequently we verify that in the weak coupling limit the critical scalar theory of section 2 is in fact a metastable scalar theory on de Sitter background with finite lifetime increasing exponentially as the coupling constant decreases. In section 4 we generalize our model to scalar theory coupled to U(1) gauge field. Such a generalization is essential as it shows how by optical observations people living in a flat Euclidean space can view a de Sitter geometry for their universe.

2 Critical scalar theory in $D = 4$ Euclidean space

In this section we study scalar field theories in four dimensional Euclidean space invariant under rescaling transformation. By rescaling we mean a coordinate transformation $x \rightarrow x' = \lambda x$, $\lambda > 0$. Requiring the kinetic term of a scalar theory to be invariant under rescaling, one verifies that the scalar field should be transformed as $\phi(x) \rightarrow \phi'(x') = \lambda^{-1}\phi(x)$. A scale free theory by definition is a theory given by an action S invariant under rescaling. In addition to the Kinetic term which variation is a total derivative, we search for polynomials $V(\phi)$ in ϕ as the potential term such that $\delta V = 0$, up to total derivatives. Such polynomials exist only in three, four and six dimensions. In the case of our interest i.e. $D = 4$,

$V(\phi) = \frac{-g}{4}\phi^4$. Here g , the coupling constant is some arbitrary real constant which is by construction invariant under rescaling. Such a scalar model is called critical as it is scale-free and its correlation length is infinite. The corresponding Euler-Lagrange equation of motion is a nonlinear Laplace equation $\nabla^2\phi + g\phi^3 = 0$. One can easily show that for $g > 0$, a solution of the non-linear Laplace equation is [1, 9],

$$\phi_0(x; \beta, a^\mu) = \sqrt{\frac{8}{g}} \frac{\beta}{\beta^2 + (x-a)^2}, \quad (1)$$

where $(x-a)^2 = \delta_{\mu\nu}(x-a)^\mu(x-a)^\nu$. β and a^μ are undetermined parameters describing the size and location of ϕ_0 . These moduli are consequences of symmetries of the action i.e. invariance under rescaling and translation. The information geometry of the moduli space, given by Hitchin formula [10]

$$\mathcal{G}_{IJ} = \frac{1}{N} \int d^4x \mathcal{L}_0 \partial_I (\log \mathcal{L}_0) \partial_J (\log \mathcal{L}_0), \quad (2)$$

is an Euclidean AdS₅ space [1],

$$\mathcal{G}_{IJ} d\theta^I d\theta^J = \frac{1}{\beta^2} (d\beta^2 + da^2). \quad (3)$$

$I = 1, \dots, 5$ counts space directions of moduli space $\theta^I \in \{\beta, a^\mu\}$, $N = \frac{4^3}{5} \int d^4x \mathcal{L}_0$ is a normalization constant and $\mathcal{L}_0 = \frac{g}{4}\phi_0^4$ is the Lagrangian density calculated at $\phi = \phi_0$. The moduli a^μ are present since the action is invariant under transformation. The existence of β is the result of invariance under rescaling. To see this let us consider scale-free fields i.e. those fields that satisfy the relation $\delta_\varepsilon\phi = 0$. Here $\delta_\varepsilon\phi(x) = \phi'(x) - \phi(x)$ is the infinitesimal scale transformation given by $\lambda = 1 + \varepsilon$ for some infinitesimal ε . To this aim we first note that rescaling leaves the origin ($x = 0$) invariant. Consequently $\phi(0)$ is distinguished from the values of the field at the other points since $\phi(0) \rightarrow \phi'(0) = \lambda^{-1}\phi(0)$. Therefore, it is plausible to make the dependence of scalar fields on their values at the origin explicit and represent the rescaling transformation by $\phi(x; \phi(0)) \rightarrow \phi'(x; \phi(0)) = \lambda^{-1}\phi(\lambda^{-1}x; \lambda\phi(0))$. Defining $\beta^{-1} = \phi(0)$, one can show that $\delta_\varepsilon\phi(x) = -\varepsilon(1 + x^I\partial_I)\phi(x)$, where $x^I \in \{x^\mu, \beta\}$. The $SO(4)$ invariant solutions of equation $\delta_\varepsilon\phi = 0$ satisfying the condition $\phi(0; \phi(0)) = \phi(0)$ are $\phi_k = c\beta^{-1} \left(\frac{\beta}{\sqrt{\beta^2 + x^2}} \right)^{k+2}$ where c is some arbitrary constant. It is easy to see that for $c = \sqrt{\frac{8}{g}}$, ϕ_0 , among the others, is the solution of classical equation of motion.

Now it is time to show that ϕ_0 is a metastable local minima of the action. Since ϕ_0 is a solution of Euler-Lagrange equation of motion $\delta S = 0$ it is a local extremum of the action³. So it is enough to show that there are field variations

³Of course from $\delta S = 0$ we can only conclude that ϕ_0 is a stationary point and not necessarily a local extremum. We continue by assuming that ϕ_0 is a local extremum. This assumption can be proved following the results of section 3.

$\phi_0 \rightarrow \phi_\eta = \phi_0 + \varepsilon\eta$ for \mathcal{C}^1 functions η vanishing as $x \rightarrow \infty$ such that $\delta S = c_\eta \varepsilon^2 + \mathcal{O}(\varepsilon^3)$ for some real positive constant c_η . For simplicity one can assume $\eta = \left(\frac{1}{1+x^2} \right)^n$, $g = 8$, $b = 1$ and calculate $\delta S = S[\phi_\eta] - S[\phi_0]$ for some integers n . One recognizes that $c_n > 0$ for $n > 5$, though it is negative for $0 < n \leq 5$. A good sign for metastability of the action at ϕ_0 . Section 3 provides an exact proof for this claim. Another interesting observation is that bubbles with larger size are less stable than those with smaller size. This can be checked noting that the size of a bubble is proportional to β^{-1} . By repeating the above calculations one easily verifies that for example for $b = 3$ $c_n > 0$ even for $n = 3$. Unfortunately without analytic data all these observations make only a rough picture of the phenomena which can not be used to make an exact statement.

The weak coupling limit

To study the weak coupling limit of the theory one can scale $g \rightarrow \frac{g}{\lambda^2}$ and consider the $\lambda \rightarrow \infty$ limit, while keeping fields ϕ undistorted. To this aim we do the following transformations,

$$\begin{aligned} x &\rightarrow \lambda x, & x &\rightarrow x, \\ \phi &\rightarrow \lambda^{-1}\phi, & \text{and} & \phi \rightarrow \lambda\phi, \\ g &\rightarrow g, & & g \rightarrow \lambda^{-2}g, \end{aligned} \quad (4)$$

which results in the desired transformation:

$$\begin{aligned} x &\rightarrow \lambda x, \\ \phi &\rightarrow \phi, \\ g &\rightarrow \lambda^{-2}g, \\ S &\rightarrow \lambda^2 S, \end{aligned} \quad (5)$$

More explicitly due to invariance under rescaling we have,

$$\int D[\phi] e^{-\frac{1}{\hbar}S[\phi; \lambda^{-2}g]} = \int D[\phi] e^{-\frac{\lambda^2}{\hbar}S[\phi; g]}. \quad (6)$$

To calculate the partition function one can instead of the transformation $S \rightarrow \lambda^2 S$ assume that $S \rightarrow S$ but $\hbar \rightarrow \lambda^{-2}\hbar$.

Consequently the $g \rightarrow +0$ limit is equivalent to $\hbar \rightarrow +0$ limit⁴. Therefore in the weak coupling limit the partition function picks up contribution only from trajectories close to ϕ_0 . This key observation when we study coupling to $U(1)$ gauge field proves why people living in a flat Euclidean space with a critical scalar field at $g = 0^+$ observe a de Sitter universe.

⁴Of course we are also blowing our universe as $x \rightarrow \lambda x$. Since the flat Euclidean space we considered is not compact it does not seem to cause any problem at this level. In the case n -point functions one should note that non-coincident points go to infinity with respect to each other

3 ϕ_0 as a de Sitter background

In this section we show that fluctuations around ϕ_0 are governed by a scalar theory on de Sitter background. This section is a review of the calculations made in [1]. In order to study the fluctuations around ϕ_0 one should rewrite the action

$$S[\phi] = \int d^4x \left(\frac{1}{2} \delta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{g}{4} \phi^4 \right), \quad (7)$$

in terms of new fields $\tilde{\phi} = \phi - \phi_0$. In this way one obtains a new action,

$$S[\phi] = S[\phi_0] + S_{\text{free}}[\tilde{\phi}] + S_{\text{int}}[\tilde{\phi}], \quad (8)$$

where $S[\phi_0] = \int d^4x \mathcal{L}_0 = \frac{8\pi^2}{3g}$, and

$$S_{\text{free}}[\tilde{\phi}] = \int d^4x \left(\frac{1}{2} \delta^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} + \frac{1}{2} M^2(x) \tilde{\phi}^2 \right), \quad (9)$$

in which,

$$M^2(x) = -3g\phi_0^2 = -24 \frac{\beta^2}{(\beta^2 + (x-a)^2)^2}. \quad (10)$$

The mass dependent term can be interpreted as interaction with ϕ_0 background. Now recall that in general, by inserting $\tilde{\phi} = \sqrt{\Omega} \bar{\phi}$ and $\delta_{\mu\nu} = \Omega^{-1} g_{\mu\nu}$ in the action $S[\tilde{\phi}] = \int d^4x \frac{1}{2} \delta^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi}$, one obtains,

$$S[\tilde{\phi}] = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \bar{\phi} \partial_\nu \bar{\phi} + \frac{1}{2} \xi R \bar{\phi}^2 \right), \quad (11)$$

i.e. a scalar theory on conformally flat background given by the metric $g_{\mu\nu}$ in which $\Omega > 0$ is an arbitrary \mathcal{C}^∞ function. R is the scalar curvature of the background and $\xi = \frac{1}{6}$ is the conformal coupling constant. For details see [11] or appendix C of [1]. Thus, defining $\bar{\phi} = \Omega^{-\frac{1}{2}} \tilde{\phi}$, one can show that $S_{\text{free}}[\tilde{\phi}]$ given in Eq.(9) is the action of the scalar field $\bar{\phi}$ on some conformally flat background with metric $g_{\mu\nu} = \Omega \delta_{\mu\nu}$:

$$S_{\text{free}}[\tilde{\phi}] = \int d^4x \sqrt{|g|} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \bar{\phi} \partial_\nu \bar{\phi} + \frac{1}{2} (\xi R + m^2) \bar{\phi}^2 \right). \quad (12)$$

Here, $m^2 \Omega = M^2(x)$, where m^2 is the mass of $\bar{\phi}$ (undetermined) and $M^2(x)$ is given in Eq.(10). This result is surprising as one can show that the Ricci tensor $R_{\mu\nu} = \Lambda g_{\mu\nu}$, where $\Lambda = -\frac{m^2}{2} > 0$ as far as $\Omega > 0$. Consequently $\bar{\phi}$ lives in a four dimensional de Sitter space which scalar curvature $R = -2m^2$. The interacting part of the action, $S_{\text{int}}[\tilde{\phi}] =$

$\int d^4x \sqrt{|g_{\mu\nu}|} \mathcal{L}_{\text{int}}$ is well-defined in terms of $\bar{\phi}$ on the corresponding dS_4 :

$$\mathcal{L}_{\text{int}} = -g \sqrt{\frac{-m^2}{3g}} \bar{\phi}^3 - \frac{g}{4} \bar{\phi}^4. \quad (13)$$

Interestingly after a shift of the scalar field $\bar{\phi} \rightarrow \bar{\phi} - \sqrt{\frac{-m^2}{3g}}$ the action (8) can be written in the dS_4 as follows:

$$S[\bar{\phi}] = \int d^Dx \sqrt{|g|} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \bar{\phi} \partial_\nu \bar{\phi} + \frac{1}{2} \xi R \bar{\phi}^2 - \frac{g}{4} \bar{\phi}^4 \right). \quad (14)$$

This is a scalar theory in a de Sitter background with reversed Mexican hat potential. $\bar{\phi} = 0$ corresponds to the local minima of the potential which distance to the center of the hill (the location of ϕ_0) is $\sqrt{\frac{\xi R}{g}}$. The height of the hill is $\frac{\xi^2 R^2}{4g}$. The lifetime of the metastable vacua can be estimated using the WKB method: the transition rate Γ is

$$\log \Gamma \sim -\Delta S, \quad \Delta S \sim V_4 \frac{(\xi R)^2}{g} \quad (15)$$

in which $V_4 \sim R^{-2}$ is the volume of the \mathcal{S}^4 , the Euclidean de Sitter space. Consequently the lifetime $\tau \sim e^{g^{-1}}$.

4 Critical scalar theory coupled to $U(1)$ gauge field

In this section we study complex critical scalar theory on Euclidean space coupled to $U(1)$ gauge field A_μ ,

$$S = \int d^4x \left(|D_\mu \phi|^2 - \frac{g}{2} |\phi|^4 \right) + S_A, \quad (16)$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative and $S_A = -\frac{1}{4} F_{\mu\nu} F_{\rho\sigma} \delta^{\rho\mu} \delta^{\sigma\nu}$. It is easy to verify that $A_\mu = 0$ and $\phi = \phi_0$ is a solution of the Euler-Lagrange equation of motion. Inserting $\tilde{\phi} = \phi - \phi_0$ in the Eq.(16), one obtains

$$S = S[\phi_0] + \tilde{S}[\tilde{\phi}] + S_{\text{int}} + S_A, \quad (17)$$

where

$$S_{\text{int}} = \int d^4x \delta^{\mu\nu} \left[(ieA_\mu (\tilde{\phi} + \phi_0) \partial_\nu (\tilde{\phi}^* + \phi_0) + c.c.) + e^2 A_\mu A_\nu |\tilde{\phi} + \phi_0|^2 \right], \quad (18)$$

$$\tilde{S}[\tilde{\phi}] = \int d^4x \left(\frac{1}{2} |\partial_\mu (\tilde{\phi} + \phi_0)|^2 - \frac{g}{4} |\tilde{\phi} + \phi_0|^4 \right). \quad (19)$$

Inserting $\bar{\phi} = \Omega^{-\frac{1}{2}} \tilde{\phi}$ where Ω is defined as before by the relation $\phi_0 = \sqrt{\frac{-m^2}{3g}} \Omega$ one obtains,

$$S_{int} = \int d^4x \frac{1}{2} \Omega \delta^{\mu\nu} \left[ieA_\mu \left(\bar{\phi} + \sqrt{\frac{-m^2}{3g}} \right) \partial_\nu \bar{\phi}^* + c.c. \right. \\ \left. + e^2 A_\mu A_\nu \left| \bar{\phi} + \sqrt{\frac{-m^2}{3g}} \right|^2 \right], \quad (20)$$

and

$$\tilde{S}[\tilde{\phi}] = \int d^4x \left(\frac{1}{2} \Omega |\partial_\mu \tilde{\phi}|^2 - \frac{g}{4} \left| \tilde{\phi} + \sqrt{\frac{-m^2}{3g}} \right|^4 \right. \\ \left. - \frac{1}{2} \sqrt{\Omega} \nabla^2 \sqrt{\Omega} \left| \tilde{\phi} + \sqrt{\frac{-m^2}{3g}} \right|^2 \right), \quad (21)$$

where $\nabla^2 = \delta^{\mu\nu} \partial_\mu \partial_\nu$. Defining $g_{\mu\nu} = \Omega \delta_{\mu\nu}$ and noting that $-\sqrt{\Omega} \nabla^2 \sqrt{\Omega} = \sqrt{g} \xi R$, after an obvious shift $\tilde{\phi} \rightarrow \tilde{\phi} - \sqrt{\frac{-m^2}{3g}}$, one obtains,

$$S = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} D_\mu \tilde{\phi} D_\nu \tilde{\phi}^* + \frac{1}{2} \xi R |\tilde{\phi}|^2 - \frac{g}{4} |\tilde{\phi}|^4 \right) \\ + S_A, \quad (22)$$

It is known that in $D = 4$, S_A is invariant under conformal transformation $\delta_{\mu\nu} \rightarrow \Omega \delta_{\mu\nu}$ and $A_\mu \rightarrow A_\mu$ thus one can write the action S_A equivalently as follows,

$$S_A = -\frac{1}{4} \int d^4x \sqrt{g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}, \quad (23)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength in the de Sitter background. Consequently at $g = 0^+$ the theory given by the action (16) is a conformally coupled scalar theory minimally coupled to $U(1)$ gauge field on de Sitter background. Therefore at $g = 0^+$ using optical instruments people living on flat Euclidean space observe an accelerating universe.

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