TTMP vol 3 (2), 2026
doi: 10.30511/TTMP.2026.2089540.1077

Transactions in Theoretical and Mathematical Physics

Observing an Open FRW de Sitter Universe Living in a Minkowski Spacetime

Farhang Loran[1 a]

[1] Department of Physics, Isfahan University of Technology, Isfahan 84156-83111, Iran

Received: 20 May 2026 / Accepted: 25 May 2026 / Published: 27 May 2026

Abstract We show that people living in a four dimensional Minkowski spacetime and located in the Fubini vacua of an unstable critical scalar theory, observe an open FRW de Sitter universe.

Keywords: critical scalar theory; Fubini vacua; de Sitter universe

1 Introduction

The WMAP results [1] combined with earlier cosmological observations shows that we are living in an accelerating universe. The currently observed lumpiness in the temperature of the cosmic microwave background is just right for a flat universe though there are also some evidences that our universe is spatially open [2]. The great simplifying fact of cosmology is that the universe appears to be homogeneous and isotropic along a preferred set of spatial hypersurfaces [3]. Of course homogeneity and isotropy are only approximate, but they become increasingly good approximations on larger length scales, allowing us to describe spacetime on cosmological scales by the Robertson-Walker metric. Constructing four dimensional de Sitter vacuum as a string theory (M-theory) solution has been a long standing challenge. An outstanding example of string theory models of de Sitter vacua are the KKLT models [4] with an exponentially large number of stable and metastable vacua without supersymmetry or with $\mathcal{N} = 1$ supersymmetry in four dimensions, the "landscape" [5]. In KKLT models, metastable de Sitter vacua of type IIB string theory is constructed by adding $\overline{\mathrm{D3}}$-branes to the GKP [6] model of highly warped IIB compactifications with nontrivial NS and RR three-form fluxes after certain fine tuning of the fluxes.

Recently, we realized that it is possible to observe a de Sitter universe while living in a flat background. This proposal was the consequence of a simple observation: the fluctuations of the scalar field around the classical trajectory of an unstable massless ϕ^4 model in four dimensional flat Euclidean spacetime is governed by a conformally coupled scalar field theory in four dimensional de Sitter background [7]. This classical trajectory is the Fubini vacua of the classically conformal-invariant scalar field theories. In [8] S. Fubini verified that critical scalar theories possess a classical vacua with $O(D,1)$ symmetry in which the expectation value of the scalar field is non-vanishing. The motivation to study such a classical vacua at that time was "to introduce a fundamental scale of hadron phenomena by means of dilatation non-invariant vacuum state in the frame work of a scale invariant Lagrangian field theory" [8]. This result is interesting due to its uniqueness. In four dimensions, in principle, one can consider two classes of critical (classically scale-free) scalar field theories i.e. massless ϕ^4 models on Euclidean spacetime with g, the coupling constant, either positive or negative (we assume the potential $V(\phi) = -\frac{g}{4}\phi^4$). Although scalar theory with $g > 0$ seems to be not physical as the potential is not bounded from below but in this case, the Euler-Lagrange equation of motion has an interesting classical solution say ϕ_0 with finite action $S[\phi_0] \sim g^{-1}$. For $g < 0$, one can still consider a solution like ϕ_0 obtained by an analytic continuation from $g > 0$ to $g < 0$ region. But such a solution is singular on the surface of a sphere which radius is proportional to g. Consequently the action $S[\phi_0]$ is infinite and ϕ_0 can not be considered as a classical trajectory. For $g > 0$ it is shown in [7] that the information geometry of the moduli space of ϕ_0 given by Hitchin formula [9] is Euclidean AdS$_5$,

$$\mathcal{G}_{IJ} d\theta^I d\theta^J = \frac{1}{\beta^2}\left(d\beta^2 + da^2\right), \tag{1}$$

[a] loran@iut.ac.ir

where $\theta^I \in \{\beta, a^\mu\}$ and $I = 1, \cdots, 5$. The moduli here are a_μ's the location of the center of ϕ_0 and β which is proportional to the inverse of the size of ϕ_0. This resembles the information geometry of SU(2) instantons. In addition $V(\phi_0)$ can be shown to be proportional to the SU(2) one-instanton density [10]. Interestingly, ϕ_0 is the bulk to boundary propagator in the AdS_5 geometry of the moduli space.

In ref. [11] we generalized the model to scalar theory coupled to U(1) gauge field. Such a generalization is essential as it shows how by optical observations people living in a flat Euclidean space observe a de Sitter geometry for their universe. In this model the massless scalar field is charged though we have not observed light charged scalars. This problem can be resolved noting that as we will show, in this model observations are made in a de Sitter background in which the scalar field appears to be conformally coupled to the de Sitter background. Therefore its mass is proportional to the scalar curvature R of the observed universe. Using the WKB approximation, the lifetime of the observed de Sitter background is calculated in [11] and is shown to be proportional to $e^{g^{-1}}$. (To my knowledge, this result is given for the first time in a beautiful paper by Coleman where ϕ_0 is called a "bounce" [12].) Consequently in the weak coupling limit $g \to {}^+0$ the lifetime increases exponentially.

In this paper we study the critical scalar theory on Minkowski spacetime. Here ϕ_0 is singular on a hyperbola in the timelike region which asymptotes to the lightcone. The total energy of ϕ_0, measured by an observer located at the center of ϕ_0 is conserved and vanishing though the energy density is a function of space and time. The energy density diverges in the neighborhood of singularity causing a gravitational collapse when the scalar theory is coupled to gravity. Fortunately the singularity is safe. On the one hand in Minkowski spacetime, the distance between the observers and the singularity is proportional to β. On the other hand there is some mechanism of β transition in the model: larger ϕ_0's decay to smaller ones due to say thermal fluctuations around ϕ_0 and finally there remains only a gas of zero sized bubbles which corresponds to $\beta \to \infty$. The mechanism of such transition is not clear yet but its phenomenology, probably is similar to that of the discretuum of possible de Sitter vacua in KKLT models [13]. Therefore, for the most stable ϕ_0 solution, the singularity is located at infinite future and is out of reach. Furthermore, from the observers point of view, the observable universe is an open de Sitter space which horizon is located on the singularity. Therefore they do not see the singularity at all for any value of β! Of course they should feel some back reactions when the scalar theory is coupled to gravity caused by the β transition. The energy density ρ and pressure p that they measure are constants satisfying the dark energy equation of state $\rho = -p = \Lambda$ ($8\pi G = 1$), where Λ is the cosmological constant. A question here is the value of Λ or equivalently R, the curvature scalar. At classical level, R is not determined in the critical scalar model as is expected. Because the theory is classically scale free. The quantum theory is not scale free due to loop corrections. Therefore quantum corrections are the hopeful candidates to give the value of the observed scalar curvature. The details are not clear for us yet and we postpone it to future works.

In Minkowski spacetime in the case of critical scalar model with negative coupling constant the singularity of ϕ_0 is a hyperbola in the spacelike region which asymptotes to the lightcone. One can easily verify that in this case the total energy for existence of ϕ_0 is infinite. Thus, similar to the Euclidean case, one can conclude that ϕ_0 uniquely exists only in the unstable ($g > 0$) critical scalar model.

The organization of the paper is as follows. In the next section we study the critical scalar theory on flat Euclidean background and determine the role of the moduli β in the stability of the solutions. We show that by recasting the scalar theory in terms of new fields $\tilde{\phi} = \phi - \phi_0$ at the end of the day one obtains a ϕ^4 model conformally coupled to a de Sitter background. In section 3 we switch to the Minkowski spacetime by a Wick rotation $t \to it$ and study the observed de Sitter universe in terms of the Robertson-Walker metric.

2 Critical scalar theory in $D = 4$ Euclidean space

In this section we study scalar field theories in four dimensional Euclidean space invariant under rescaling transformation $x \to x' = \lambda x$, $\lambda > 0$. There are in general three scale-free scalar theories: ϕ^4 model in $D = 4$ and ϕ^3 and ϕ^6 models in $D = 6, 3$ respectively. In this paper we only study ϕ^4 model in $D = 4$ though the main result of this paper can be simply generalized to the other two scalar models. The action in Euclidean space is

$$S[\phi] = \int d^4x \left(\frac{1}{2}\delta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{g}{4}\phi^4 \right) \tag{2}$$

where we assume $g > 0$. Consequently the potential $V(p) \sim -\phi^4$ and is not bounded from below. The Kronecker delta symbol $\delta^{\mu\nu}$ stands for the metric of flat Euclidean space. The corresponding equation of motion is a non-linear Laplace equation $\nabla^2\phi + g\phi^3 = 0$, where $\nabla^2 = \delta^{\mu\nu}\partial_\mu\partial_\nu$. One can easily show that for $g > 0$, a solution of the non-linear Laplace equation is

$$\phi_0(x;\beta,a^\mu) = \sqrt{\frac{8}{g}}\frac{\beta}{\beta^2 + (x-a)^2}, \tag{3}$$

where $(x-a)^2 = \delta_{\mu\nu}(x-a)^\mu(x-a)^\nu$. β and a^μ are undetermined parameters describing the the size and location of ϕ_0. These moduli are consequences of symmetries of the action i.e. invariance under rescaling and translation. The information geometry of the moduli space, given by Hitchin formula [9]

$$\mathcal{G}_{IJ} = \frac{1}{N}\int d^4x \mathcal{L}_0 \partial_I (\log \mathcal{L}_0)\, \partial_J (\log \mathcal{L}_0), \tag{4}$$

is an Euclidean AdS_5 space (1). $N = \frac{4^3}{5}\int d^4x \mathcal{L}_0$ is a normalization constant and $\mathcal{L}_0 = \frac{g}{4}\phi_0^4$ is the Lagrangian density calculated at $\phi = \phi_0$. The moduli a^μ are present since the action is invariant under translation. The existence of β is the result of invariance under rescaling [11].

To my knowledge, the solution ϕ_0 is obtained for the first time by Fubini. He looked for a solution of the equation of motion "in which the vacuum expectation value of the field $\phi(x)$ is non-vanishing" [8]. He verified that this vacua is not invariant under the Poincare group but is invariant under the de Sitter group $O(3,1)$. Consequently by recasting the action in terms of new fields $\tilde{\phi} = \phi - \phi_0$ one expects to obtain, after some field redefinitions, a scalar theory in de Sitter background. In fact the action in terms of $\tilde{\phi}$ is,

$$S[\phi] = S[\phi_0] + S_{\text{free}}[\tilde{\phi}] + S_{\text{int}}[\tilde{\phi}], \tag{5}$$

where $S[\phi_0] = \int d^4x \mathcal{L}_0 = \frac{8\pi^2}{3g}$, and

$$S_{\text{free}}[\tilde{\phi}] = \int d^4x \left(\frac{1}{2}\delta^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} + \frac{1}{2}M^2(x)\tilde{\phi}^2\right), \tag{6}$$

in which,

$$M^2(x) = -3g\phi_0^2 = -24\frac{\beta^2}{(\beta^2 + (x-a)^2)^2}. \tag{7}$$

These equations show that ϕ_0 is a metastable local minima of the action. This can also be verified explicitly by numerical analysis of action (2), see ref. [11]. Equation (6) can be used to show that the stability increases as $\beta \to \infty$. In fact if we calculate the variation of action at the stationary point $\phi_0(\beta)$ for different values of the moduli β_1 and β_2, under variation $\delta\phi$, from Eqs.(6,7) one verifies that,

$$\begin{aligned}\Delta S &= \delta S|_{\beta_1} - \delta S|_{\beta_2} \\ &\sim \int d^4x \left(\phi_0(\beta_2)^2 - \phi_0(\beta_1)^2\right)\delta\phi^2 + \mathcal{O}(\delta\phi^3).\end{aligned} \tag{8}$$

For simplicity we assume that $a_i^\mu = 0, i = 1,2$. Therefore ΔS is proportional to,

$$(\beta_1^2 - \beta_2^2)\int_0^\infty dx \frac{x^3(-x^4 + \beta_1^2\beta_2^2)}{(\beta_2^2 + x^2)^2(\beta_1^2 + x^2)^2}\delta\phi^2. \tag{9}$$

For $\delta\phi$ with compact support, i.e. $\delta\phi = 0$ *if* $|x| > \sqrt{\beta_1\beta_2}$ the integral above is positive therefore $\Delta S \sim (\beta_1^2 - \beta_2^2)$. As far as ϕ_0 is a metastable local minima there exist $\delta\phi$ with compact support such that $\delta S|_{\beta_i} > 0$ $i = 1,2$. Consequently if $\beta_1 > \beta_2$ then $\delta S|_{\beta_1} > \delta S|_{\beta_2} > 0$. One can convince herself/himself that for some $\delta\phi$ one obtains $\delta S|_{\beta_2} < 0$ while $\delta S|_{\beta_1} > 0$. Consequently one concludes that there is a transition $\beta_2 \to \beta_1$ induced by say, thermal fluctuations. In addition the stability increases as $\beta \to \infty$.

The mass term in Eq.(6) can be interpreted as interaction with the background ϕ_0. Now recall that in general, by inserting $\tilde{\phi} = \sqrt{\Omega}\bar{\phi}$ and $\delta_{\mu\nu} = \Omega^{-1}g_{\mu\nu}$ in the action $S[\tilde{\phi}] = \int d^4x \frac{1}{2}\delta^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}$, one obtains,

$$S[\tilde{\phi}] = \int d^4x \sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} + \frac{1}{2}\xi R\bar{\phi}^2\right), \tag{10}$$

i.e. a scalar theory on conformally flat background given by the metric $g_{\mu\nu} = \Omega\delta_{\mu\nu}$ in which $\Omega > 0$ is an arbitrary $\mathcal{C}^\infty$ function. R is the scalar curvature of the background and $\xi = \frac{1}{6}$ is the conformal coupling constant. For details see [14] or appendix C of [7]. Thus, defining $\bar{\phi} = \Omega^{\frac{-1}{2}}\tilde{\phi}$, one can show that $S_{\text{free}}[\tilde{\phi}]$ given in Eq.(6) is the action of the scalar field $\bar{\phi}$ on some conformally flat background,

$$\begin{aligned}S_{\text{free}}[\phi] = \int d^4x \sqrt{|g|}\Big(&\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi \\ &+ \frac{1}{2}(\xi R + m^2)\phi^2\Big).\end{aligned} \tag{11}$$

with metric

$$g_{\mu\nu} = \Omega\delta_{\mu\nu}, \qquad \Omega = \frac{M^2(x)}{m^2}, \tag{12}$$

where m^2 is the mass of $\bar{\phi}$ (undetermined) and $M^2(x)$ is given in Eq.(7). This result is surprising as one can show that the Ricci tensor $R_{\mu\nu} = \Lambda g_{\mu\nu}$, where $\Lambda = -\frac{m^2}{2} > 0$ as far as $\Omega > 0$. Consequently $\bar{\phi}$ lives in a four dimensional de Sitter space which scalar curvature $R = -2m^2$. The interacting part of the action, $S_{\text{int}}[\tilde{\phi}] = \int d^4x \sqrt{|g_{\mu\nu}|}\mathcal{L}_{\text{int}}$ is well-defined in terms of $\bar{\phi}$ on the corresponding dS_4:

$$\mathcal{L}_{\text{int}} = -g\sqrt{\frac{-m^2}{3g}}\bar{\phi}^3 - \frac{g}{4}\bar{\phi}^4. \tag{13}$$

Interestingly after a shift of the scalar field $\bar{\phi} \to \bar{\phi} - \sqrt{\frac{-m^2}{3g}}$ the action (5) can be written in the dS_4 as follows:

$$\begin{aligned}S[\bar{\phi}] = \int d^4x \sqrt{|g|}\Big(&\frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}\,\partial_\nu\bar{\phi} \\ &+ \frac{1}{2}\xi R\bar{\phi}^2 - \frac{g}{4}\bar{\phi}^4\Big).\end{aligned} \tag{14}$$

This is a scalar theory in a de Sitter background with reversed Mexican hat potential. In a similar way, by recasting the critical scalar theory minimally coupled to $U(1)$ gauge field in terms of fluctuations around the classical solution $\phi = \phi_0$ and $A_\mu = 0$, one verifies that the action

$$S = \int d^4x \left(|D_\mu\phi|^2 - \frac{g}{2}|\phi|^4\right) + S_A, \tag{15}$$

is equivalent to

$$S = S[\phi_0] + S[\bar{\phi}, A_\mu] + S_A, \tag{16}$$

where

$$S[\bar{\phi}, A_\mu] = \int d^4x \sqrt{g} \left(\frac{1}{2} g^{\mu\nu} D_\mu \bar{\phi} D_\nu \bar{\phi}^* + V(\bar{\phi}) \right), \tag{17}$$

$V(\bar{\phi}) = \frac{1}{2}\xi R \left|\bar{\phi}\right|^2 - \frac{g}{4}\left|\bar{\phi}\right|^4$ and S_A is the Kinetic term for the gauge field,

$$\begin{aligned} S_A &= -\frac{1}{4} \int d^4x F_{\mu\nu} F_{\rho\sigma} \delta^{\rho\mu} \delta^{\sigma\nu} \\ &= -\frac{1}{4} \int d^4x \sqrt{g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}. \end{aligned} \tag{18}$$

$F_{\mu\nu}$ in the first equality above is the field strength in Minkowski spacetime. In the second equality $F_{\mu\nu}$ should be understood as the field strength on the de Sitter space [11]. It should be noted that under the conformal transformation $g_{\mu\nu} \to \Omega g_{\mu\nu}$, in four dimensions $A_\mu \to A_\mu$.

3 The critical scalar theory in Minkowski spacetime

The critical scalar theory in four dimensional Minkowski spacetime is given by the action

$$S[\phi] = \int d^4x \left(\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{g}{4} \phi^4 \right), \tag{19}$$

where $\eta_{\mu\nu} = (+,-,-,-)$ and $g > 0$. The equation of motion is a non-linear wave equation $\eta^\mu \partial_\mu \partial_\nu \phi - g\phi^3 = 0$ which has the solution

$$\phi_0 = \sqrt{\frac{8}{g}} \frac{\beta}{\beta^2 - (t-a^0)^2 + |\vec{x} - \vec{a}|^2}, \tag{20}$$

where $\vec{x} \in \mathbf{R}^3$. Here on we assume $a^\mu = 0$ for simplicity. ϕ_0 is singular on the hyperbola $t^2 = x^2 + \beta^2$ and we define its distance to an observer located on the origin to be given by β. The Hamiltonian density $\mathscr{H}$ corresponding to ϕ_0, is,

$$\mathscr{H} = \frac{16\beta^2}{g} \frac{t^2 + x^2 - \beta^2}{(-t^2 + x^2 + \beta^2)^4} \tag{21}$$

which tends to infinity in the vicinity of the singularity. As is explained in the introduction, using the results of section 2 and the arguments after Eq.(9), we now that the most stable ϕ_0 is the zero-sized one, corresponding to $\beta \to \infty$. Therefore the singularity is safe when the scalar theory is coupled to gravity. For $t < \beta$ one can calculate, say, the total vacuum energy $H = \int d^3x \mathscr{H}$ corresponding to ϕ_0 which is surprisingly vanishing, $H = 0$. Repeating the calculations of section 2 one verifies that observers located at the origin of the Minkowski spacetime observe a de Sitter space given by the conformally flat metric,

$$ds^2 = \frac{12\beta^2}{\Lambda} \frac{1}{(\beta^2 - t^2 + x^2)^2} (-dt^2 + d\vec{x}^2), \tag{22}$$

where $\Lambda > 0$ is the cosmological constant. This metric can be obtained using Eq.(12) after a Wick rotation $t \to it$. We use a different set of coordinates in order to describe the observed de Sitter space with FRW metric to see whether it is open, closed or flat. Defining, coordinates u, ρ and z_i, $i = 1,2,3$ by the relations $z_i^2 = 1$, $t = u\cosh\rho$ and $x_i = u\sinh\rho z_i$ useful to describe the timelike region $t > |\vec{x}|$, one obtains,

$$\begin{aligned} ds^2 = \frac{12\beta^2}{\Lambda} & \frac{1}{(\beta^2 - u^2)^2} \\ & \times \left(-du^2 + u^2\left(d\rho^2 + \sinh^2\rho\, dz_i^2\right)\right). \end{aligned} \tag{23}$$

we define a time coordinate τ by the relation $d\tau = \left(\beta^2 - u^2\right)^{-1} du$. Thus one obtains,

$$\begin{aligned} ds^2 = \frac{12\beta^2}{\Lambda} \Bigg[& -d\tau^2 \\ & + \frac{\sinh^2(2\beta\tau)}{4\beta^2} \left(d\rho^2 + \sinh^2\rho\, dz_i^2\right) \Bigg]. \end{aligned} \tag{24}$$

One can call the region $u < \beta$ which can be observed by observers located on the origin the south pole and the $u > \beta$ region the north pole, a known terminology in de Sitter geometry. The south pole and north pole in our model are separated by the horizon located at $u = \beta$, i.e the singularity of ϕ_0. By normalizing τ by the normalization factor $\sqrt{\frac{12}{\Lambda}}\beta$ and defining a new coordinate $r = \sinh\rho$, one at the end of the day obtains,

$$ds^2 = -d\tau^2 + a(\tau)^2 \left(\frac{dr^2}{1+r^2} + r^2 dz_i^2 \right), \tag{25}$$

in which $a(\tau) = \sqrt{\frac{3}{\Lambda}} \sinh\sqrt{\frac{\Lambda}{3}}\tau$. This is the Robertson-Walker metric for open de Sitter universe. One can easily calculate the energy density ρ and the pressure p of the cosmological stuff corresponding to ϕ_0 using the Friedmann equations for the open universe,

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho + \frac{1}{a^2}, \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p). \end{aligned} \tag{26}$$

One verifies that p and ρ satisfy the equation of state for the cosmological constant $\rho = -p = \Lambda$ $(8\pi G = 1)$.

References

1. D.N. Spergel et al., *Astrophys. J. Suppl.* **148** (2003) 175, DOI: 10.1086/377226, arXiv: astro-ph/0302209.
2. J.R. Gott, Nature 295 (1982) 304, DOI: 10.1038/295304a0.
3. P.J. Peebles, D.N. Schramm, E.L. Turner and R.G. Kron, Nature 352 (1991) 769.

4. S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *Phys. Rev. D* **68** (2003) 046005, DOI: 10.1103/PhysRevD.68.046005, arXiv: hep-th/0301240.
5. L. Susskind, *The Anthropic Landscape of String Theory*, arXiv: hep-th/0302219.
6. S.B. Giddings, S. Kachru and J. Polchinski, *Phys. Rev. D* **66** (2002) 106006, DOI: 10.1103/PhysRevD.66.106006, arXiv: hep-th/0105097.
7. F. Loran, Phys. Rev. D 71 (2005) 126003, DOI: 10.1103/PhysRevD.71.126003, hep-th/0501189.
8. S. Fubini, Nuovo Cim. A34 (1976) 521.
9. N. J. Hitchin, *The Geometry and Topology of Moduli Spaces* in *Global Geometry and Mathematical Physics*, (Springer, Heidelberg, 1988) 1–48;
   R. Britto, B. Feng, O. Lunin and S. J. Rey, *Phys. Rev. D* **69** (2004) 126004, DOI: 10.1103/PhysRevD.69.126004, arXiv: hep-th/0311275;
   S. Parvizi, *Mod. Phys. Lett. A* **17** (2002) 341–354, DOI: 10.1142/S0217732302006568, arXiv: hep-th/0202025.
10. M. Blau, K. S. Narain and G. Thompson, *Instantons, the Information Metric, and the AdS/CFT Correspondence*, arXiv: hep-th/0108122.
11. F. Loran and E. Bavarsad, “Metastable de Sitter Vacua from Critical Scalar Theory,” *Trans. Theor. Math. Phys.* (2026), 54–58. DOI: 10.30511/ttmp.2026.2089573.1078, .
12. S.R. Coleman, Phys. Rev. D15 (1977) 2929, DOI: 10.1103/PhysRevD.15.2929.
13. A.R. Frey, M. Lippert and B. Williams, *Phys. Rev. D* **68** (2003) 046008, DOI: 10.1103/PhysRevD.68.046008, arXiv: hep-th/0305018.
14. T. Jacobson, *Introduction to Quantum Fields in Curved Spacetime and the Hawking Effect*, gr-qc/0308048.