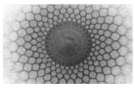
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Bound State of an Electron on a ^{4}He Superfluid Droplet as a Test for Quantum Gravity

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Abstract We address the problem of finding a system in which there would be measurable quantum gravitational effects. Following standard quantum-field methods, we have calculated the first-order radiative correction of graviton exchange on the binding energy of an electron with an ultra cold superfluid droplet of ^{4}He with mass about the Planck mass. For two ^{4}He droplets with a mass difference of about one microgram, we show that the relative difference in the binding energies is about one percent.

Keywords: Bound State; ^{4}He Superfluid Droplet; Test for Quantum Gravity

1 Introduction

The most important obstacle in the search for quantum gravity (QG) is the absence of the experimental input that is needed to test the theory. There are papers on the phenomenology of quantum gravity [1] but the magnitude of the proposed QG effects are expected to be negligible in standard. circumstances because the Planck scale suppresses them. However, small effects can become observable in special contexts. and one can always search for an experimental setup such that a very large number of the very small quantum-gravity contributions are effectively summed together. For instance, the ratio of the forces of gravity and electricity between two particles with mass m and charge e is

$$\frac{F_g}{F_e} = \frac{Gm^2}{e^2} = \frac{hc}{2\pi e^2}\left(\frac{m}{M_{\text{Planck}}}\right)^2, \tag{1}$$

that is too small for the fundamental particles with the ordinary masses. It is well known that electrons can be bound in surface states. outside certain bulk dielectrics such as ^{4}He [2]. Experimentally, the binding energy of the electron to the surface of liquid helium has been measured using millimeter-wave spectroscopy to be 8 Kelvin [3]. R. Y. Chiao by replacing the oil of the classic "Millikan oil drops" with superfluid helium (^{4}He) with a gravitational mass of around the Planck-mass scale ($M_{\text{Plank}} \approx 22$ micrograms), proposed an experimental setup for a gravity-wave antenna [4]. In this Letter we propose another experiment to detect the interplay between gravity and QED by exploring the bound state of an electron to an ultra cold superfluid drop of ^{4}He (e-^{4}He bound state).

2 Gravitational Quantum Corrections to the Binding Energy

The e-^{4}He bound state is due to formation of an image charge inside the dielectric sphere of the drop. The image potential is of the form [5]

$$V = -Ze^2/r, \tag{2}$$

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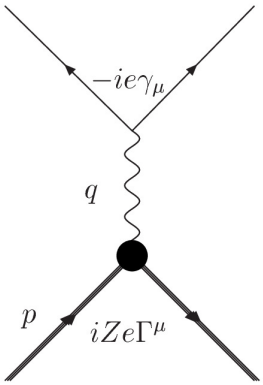


Fig. 1 One photon exchange diagram for e-^{4}He bound state. The upper legs show the electronic current, while the lower legs stand for the ^{4}He drop current.

where r is the radial distance of the electron from the droplet surface and

$$Z = \frac{(k-1)}{(k+1)}, \tag{3}$$

in which $k \approx 1.057$ is the relative permittivity of Helium. Because there is a barrier of 1 eV, the penetration of the electron with binding energy of a few Kelvin, into the Helium drop is negligible. Thus, one can assume the e-^{4}He bound state as an ideal Hydrogen-like system. Using Eq.(3), one can see that the binding energy is about 8 Kelvin ($\approx 10^{-3}$ eV) and the effective Bohr radius $r_{\rm B} = 76$ [6]. This justifies the tick-film approximation we have considered to obtain Eq.(3) as far as the radius of the Helium drop is about one millimeter.

In QED, the tree-level diagram corresponding to the hydrogen-like bound state is give by Fig.(1).

In this letter, we would like to explore the QG effects in the e-^{4}He bound state. As is seen from Eq.(1), classically, only for particles with mass comparable to the Planck mass the gravitational force is comparable with the electrical force. Therefore, at tree level, the one graviton exchange diagram between the electron and ^{4}He is negligible in comparison with the corresponding one-photon exchange diagram. In contrast, one anticipates that for the one loop corrections, the QG-correction for the photon-^{4}He vertex to be the only significant term in comparison with the other one loops coming from both QED and QG, see Fig.(2). Thus,

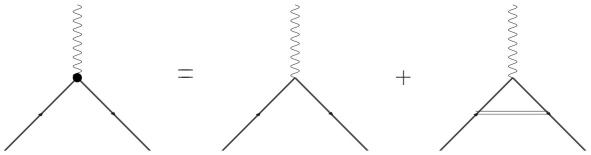


Fig. 2 The most significant gravitational one-loop correction for the ^{4}He-photon vertex. The double line represents the graviton.

gravitational quantum corrections can have a valuable contribution to the binding energy of e-^{4}He bound state. Consequently, this mesoscopic system seems to be a good lab for probing the nature of the interplay between gravitation and quantum mechanics.

The straightforward framework for general relativity quantized for small fluctuations is a nonrenormalizable quantum field theory. In fact, it is shown that the one-loop divergences in the theory of scalar field coupled to quantum general relativity, can only be cancelled by counterterms that do not appear in the original action [7]. Nevertheless, this framework is appropriate for describing interactions at energies and momenta below the Planck scale $M_{\rm Planck}$ when treated as an effective low-energy theory [8, 9].

To calculate the one-loop diagram in Fig.(2) we consider the action for transverse-traceless small fluctuation modes $h_{\mu\nu}$, given by the following Lagrangian,

$$\begin{aligned}\mathscr{L} = &\frac{M^2_{\rm Planck}}{128\pi}\eta^{\alpha\beta}\partial_\alpha h^{\mu\nu}\partial_\beta h_{\mu\nu} \\ &+(\eta^{\mu\nu}+h^{\mu\nu})\,\partial_\mu\phi^*\partial_\nu\phi+\cdots,\end{aligned} \tag{4}$$

in which $\eta_{\alpha\beta}$ is the Minkowski metric. In this way, for the low momentum transfer, one can show that

$$\begin{aligned}\Gamma^\mu = p^\mu + \frac{1}{\pi}\frac{m^2_{^4\rm He}}{M^2_{\rm Planck}}\Bigg\{&-\frac{11}{6}+\frac{1}{2}\ln\left(\frac{\Lambda}{m_{^4\rm He}}\right) \\ &-\ln\left(\frac{\lambda}{2m_{^4\rm He}}\right)+\mathscr{O}\left(\frac{q^2}{m^2_{^4\rm He}}\right)\Bigg\}p^\mu,\end{aligned} \tag{5}$$

Where $\Lambda \sim M_{\rm Planck}$ is the ultraviolet cutoff and λ is an infrared cutoff. It should be noted that the mass of the helium droplet is not a fundamental parameter of the quantum gravity and therefore the first three terms due to the one loop correction in Eq.(5) can not be absorbed into any properly

renormalized vertex function. In fact, the QG-structural effects can be described by a form factor which, in principle, can be measured in the e-^{4}He scattering processes for different ^{4}He droplets. The form factor at vanishing four momentum transfer for an appropriate *reference mass* m_r can be set to unity. Therefore the vertex function can be written as follows.

$$\Gamma^\mu = p^\mu + \frac{1}{\pi}\frac{m^2_{^4\mathrm{He}}}{M^2_{\mathrm{Planck}}}\left\{\ln\left(\frac{m_{^4\mathrm{He}}}{m_r}\right) + \mathscr{O}\left(\frac{q^2}{m^2_{^4\mathrm{He}}}\right)\right\} p^\mu, \quad (6)$$

3 Conclusion

From Eq.(6) one verifies that the binding energy depends on the mass of ^{4}He droplet. Therefore, the physical observable can be the difference in the binding energies of different *e*-^{4}He droplets with different masses,

$$\frac{\Delta E}{E} \simeq \frac{1}{\pi M^2_{\mathrm{Planck}}}\Delta\left(m^2_{^4\mathrm{He}}\ln\frac{m_{^4\mathrm{He}}}{m_r}\right). \quad (7)$$

where E is the binding energy of *e*-^{4}He system at the tree level and ΔE is the energy difference due to the one loop QG-correction for two ^{4}He droplets with different masses. For instance, for two ^{4}He droplets with mass difference of about one microgram, and $m_r \simeq M_{\mathrm{Planck}}$ the relative difference in the binding energies will be about two percent.

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